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On Wheeler's Notion of "Law without Law" in Physics[1]

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Wheeler's idea that physical "laws" would not appear in a truly fundamental description of nature is critically examined.

Is it possible that there is an ultimate law of physics, a principle $\mathscr{P}$ from which follows everything that is knowable about the material world? If so, what can we already infer or postulate about the form that this principle takes? Wheeler's idea of "law without law" is an attempt to begin to answer these questions.

If there were no all-explanatory physical principle $\mathscr{P}$ approachable by the methods of science, this would presumably mean that there exist aspects of the natural world that are fundamentally inaccessible to science. This would run directly counter to rationalism and to our view of physics as the universal science, which have hitherto been the driving forces behind progress in the subject and which we should be extremely reluctant to abandon.

But if $\mathscr{P}$ were itself a law of physics, then the problem of $\mathscr{P}$'s own origin—why that particular principle holds in nature rather than some other—would be forever insoluble. And hence $\mathscr{P}$ would not be all-explanatory within physics. So, paradoxically $\mathscr{P}$, the ultimate principle of physics, cannot be a "law" (of physics). Hence the expression "law without law."

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In what follows, I shall be criticizing Wheeler's actual proposals for a "law without law." But however flawed they may be, the motivating argument that I have just summarized, that $\mathscr{P}$ must exist but cannot be a physical law, seems to me to be very compelling and I shall not quarrel with it. Indeed, at the end of this article I make some suggestions of my own about what $\mathscr{P}$ must look like.

There seems to be a consensus among physicists that physics is now, for the first time, approaching the point where its fundamental theories will be truly universal in their intended scope. We hope soon to be constructing theories, not about this or that special type of physical system, as has always been the case in the past, but about the whole of material reality. Furthermore, many believe (see, e.g., Ref. 1) that nothing qualitatively new lies between us and such a unified theory of nature. There are problems, but it is widely expected that their solutions will take the same *form* as our existing theories.

Certainly quantum theory, the general framework within which all other physical theories are currently formulated, has no known experimental counterexample and no serious rival theory. Within the quantum framework, a physical theory amounts to the specification of an action functional and the quantum field configurations that constitute its domain. Thus the expectation is that we shall soon disover the ultimate "fundamental fields," their invariance properties, and their action functional. The principle of the stationarity of this action functional, together with the principles of quantum theory, would be the ultimate dynamical "laws of nature."

This consensus has been repeatedly attacked by Wheeler.[2,3]. He does not necessarily deny that we are now close to constructing universal *dynamical* theories. But he argues that by their very nature dynamical laws cannot be fundamental. The idea of an immutable dynamical law, far from being a feature we should expect in an "ultimate" description of the world, is in fact only characteristic of the temporary, incomplete descriptions we have had to resort to along the way.

For dynamical laws, at least as we have known them, are always expressible as conservation laws, or symmetry or invariance principles. And Wheeler argues that symmetry and invariance principles are always indicative of a level of structure *below* the one being described. Yet at the same time they mask the nature of this structure. For example, according to the ideal gas law

$$\frac{pV}{nT} = R$$

n moles of gas at constant volume V exert a pressure p on their container proportional to the temperature T. Superficially this law gives no clue that there is a level of description of gases deeper than that via "pressure" and "temperature." But, many years after the ideal gas law was first postulated, its derivation from kinetic theory gave us a compelling token of the existence of an underlying structure, i.e., of the existence of molecules. Even that told us very little indeed about the molecules themselves. We can learn a little more about molecular and atomic structure by measuring small deviations from the ideal gas law, but this is an uphill struggle. The very concepts of "pressure" and "temperature" are fundamentally misleading. Nature at the molecular level cannot even be described in terms of them. Thus the complex form of the underlying theory is masked, rather than revealed, by the very "law" which is the token of its presence.

An invariance principle asserts that some physical quantity is *constant*, or is *independent* of certain variable parameters. For example, the gas law says that R is independent of p, V, T, and n. This "immutability" worries Wheeler who points out that many of the quantities thought immutable in the past have later turned out to be very mutable under suitably extreme conditions. (And perhaps we should also bear in mind that this is not the first time in history that physicists have thought they were close to an all-explanatory theory.) Of course we must expect obsolete physical theories to contain obsolete invariance principles, so the fact that our old theories were wrong lends no credence to the proposition that our new ones will be too! But Wheeler is interested not so much in the demise *per se* of certain invariance principles, but in the manner of their demise of late. In the old days, a theory was abandoned when it turned out to be false; i.e., when it was refuted in favor of a truer theory, of which it became a limiting case. Ohm's law is a favorite elementary example, but for more recent cases one might cite the law of conservation of mass, which is violated in the special theory of relativity, or Euclid's parallel axiom which is violated in the general theory of relativity. But recently, some invariance principles have not really been *refuted* by the discovery of exceptions, or conflicts with better theories, but rather what Wheeler calls "*transcended*" by our increasing understanding of existing theories. A theory is transcended when it is found to be not so much less *true*, as less *fundamental*, than had previously been thought. The conservation law for baryon number, for example, was transcended when it was discovered that a black hole evaporating via the Hawking process emits equal numbers of baryons and antibaryons, even if it was originally formed entirely out of baryons. This would happen even if the divergence of the baryon current was everywhere zero in space-time. The success of the "big bang" cosmological model, and the singularity theorems of Penrose and Hawking (see Ref. 4) have led us to consider seriously the possibility that *all* our conservation laws must be transcended sooner or later, for at a space-time singularity the passage of time itself is not "immutable": time ends (or begins).

Perhaps we are indeed approaching the final truth of *dynamics*, only to find that dynamics is not, after all, fundamental in physics. What is fundamental, then? It must be conceded there are some physical problems which there is no reason to expect would be solved even if we knew the ultimate dynamical laws. These may be classified into two broad categories:

1. How is the actual solution of the dynamical equations distinguished from the infinity of possible ones?
2. Whet underlying principles require the dynamical laws to take this, and not some other form, and what principles unify the dynamical laws, the quantum framework, and the solution of (1)?

Prominently among problems of type (1) I would list

The origin of thermodynamic phenomena such as obedience to the second law of thermodynamics.

The nature of the quantum state of the universe (see, e.g., Refs. 5–7).

Determination of the boundary conditions and initial conditions on quantum fields.

And among problems of type (2):

The "factor ordering problem" in quantum theory, or the problem of finding a consistent operator formalism to replace the stopgap of "quantization."[8]

The relationship between physics and the theory of computation.[9,10]

Such problems have traditionally been regarded as secondary in importance, or even outside the scope of physics altogether. Wheeler[3] looks forward to a "third era" of physics in which they will be primary.

But Wheeler has intentions even more radical than this. It is really not just dynamics that he wants to regard as ultimately ephemeral, but the whole notion of a physical law. For suppose we were successful in solving all the type (1) and type (2) problems. Would the solution be a physical law or principle? Wheeler would say "no."

Our dilemma in contemplating the possibility of an all-explanatory physical theory $\mathscr{P}$ can be summarized thus: *Is $\mathscr{P}$ an analytic or a synthetic proposition*? An analytic proposition is one whose truth or falsity is a consequence of logic alone. A synthetic proposition is a meaningful proposition that is not analytic. Physical theories as we know them are always synthetic, because they must make testable assertions. An analytic proposition (that is true) makes no assertion about the world. It is empty. Thus, if $\mathscr{P}$ is to have testable theories as its consequences, it must itself be synthetic. On the other hand, if $\mathscr{P}$ is synthetic, it must contain some "arbitrary" postulates not deducible from anything more fundamental, so how can it be "all-explanatory"? How can it explain why these particular postulates, and not some others, are true in nature.

In the past, Wheeler and others have attempted to escape from this dilemma by seeking some fundamental mathematical structure with which the physical world could be identified, so that physical laws would have the status of theorems. Thus he[11] suggested that underlying geometry there might be a "pregeometry" with the structure of symbolic logic. Similarly Penrose[12] has suggested that "We [may all be] composed of abstract mathematics." Maybe so, but I believe that such approaches do not really address the problem of what $\mathscr{P}$ may be. For one cannot derive something synthetic (physics) from something analytic (mathematics). Some of the physical laws may be "theorems." But there will always be others which are "axioms." The entire physical content of such theories would lie in the method of identification of the physical world with the abstract mathematical structure.

Another possibility suggested by Wheeler is that underlying the physical laws there is no structure at all, just "chaos." In his words "There is no law except the law that there is no law." He had in mind the emergence of approximate lawlike behavior, such as the ideal gas law, from an underlying chaotic complexity like that of the motion of myriads of molecules. But again this proposal fails to address the problem. Lawlike behavior could never emerge from something that obeyed no laws. The motion of molecules is not lawless, it is *random*. And randomness implies a probability distribution function, and that function, or the principle from which it may be derived, is the missing underlying law.

Wheeler has recognized this deficiency in his original "order from chaos" idea, and has now modified it to

"Chaos" + "*regulating principle*" → approximate "laws"

In the case of kinetic theory, the "regulating principle" is the "principle of equal *a priori* probability" of statistical mechanics. But in making a fundamental "regulating principle" the new object of his search, Wheeler *ipso facto* abandons the search for "law without law." For a regulating principle is a law, and his modified proposal can be rewritten in the uninspiring form

"Stochastic laws" → approximate "deterministic laws"

Yet another possibility that Wheeler has investigated from time to time (e.g., Refs. 2 and 13) is that the underlying reality is composed entirely out of "individual acts of observation" (or "consciousness" or "measurement").

This sort of proposal has a long history, though not so much in physics as in antirealist philosophies of science such as subjectivism, instrumentalism, and logical positivism. I refer the reader to Popper's[14] refutations of these. Admittedly the fact that a proposed new physical theory is "philosophically untenable" should never deter a physicist: It is more likely that philosophy is wrong than physics. But I believe[15] that Wheeler's physical motivation in this case is also misguided, deriving as it does from an erroneous interpretation of quantum theory.

Nevertheless this last proposal, though hopeless in itself, has one salvageable feature which I believe really is a clue to the nature of $\mathscr{P}$: Instead of trying futilely to get "something for nothing," a synthetic proposition from an analytic, it places at the foundation of physics assertions which are synthetic (e.g. that there are "acts of observation" in the world), but which *have to be postulated anyway* for some reason outside physics.

What are these "synthetic propositions which have to be postulated anyway"? The idea that there are such propositions and that physical theories can be deduced from them is not new. Wheeler's appeal to the existence of acts of observation is also Descartes' ultimate source of justification of beliefs "*cogito ergo sum.*" And Kant made a whole collection of "synthetic *a prioris*," which were synthetic propositions that he believed were necessary for thought.

However, modern rationalist epistemology gets along without ever "justifying" beliefs; no method of justification has any authority. Nor need we nowadays accept that any synthetic proposition is "necessary for thought." After all, Kant included Euclidian geometry among his "synthetic *a prioris*," yet no physicist nowadays believes that Euclidian geometry is a true theory, let alone "necessary."

We cannot hope to find any synthetic proposition that can be proved or established in any way as necessarily true, or even probable. What we can do, however, is to examine more closely and systematically what synthetic propositions we are in fact, for whatever reason, assuming *a priori* (by which I now mean merely "prior to any physical principle"). For although there is no synthetic proposition so self-evident that we *must* assume it true, it is equally the case that we cannot make progress without making some *a priori* assumptions, even though as rationalists we should do this tentatively and always be ready to criticize and replace any one of them.

One thing that we always tacitly assume *a priori* in the search for any physical theory is that the physical process of that theory becoming known and expressed is not in itself forbidden by the theory. No physical principle that we can know can itself forbid our knowing it. That *every* physical principle must satisfy this highly restrictive property is a synthetic *a priori* proposition, not because it is *necessarily* true, but because we cannot help assuming it to be true in seeking to know the principle.

Very similar arguments motivate the "anthropic principle" in its various forms.[16] For example, it has been argued that the values of certain fundamental physical constants may be inferred with some precision from the very fact that someone exists who has asked what their values are.

It is hard to make watertight inferences from the anthropic principle because they always depend on assumptions about what physical conditions are necessary for the emergence of "someone who asks." However, the *a priori* meta-principle that "the theory we are constructing cannot say that we shall not construct it" can be expressed without anthropic references:

> *Successive approximations to every physical principle can be encoded in the states of realizable physical systems.*

Notice that this can be postulated independently of the existence of an all-explanatory principle $\mathscr{P}$. But if $\mathscr{P}$ exists, this meta-principle is surely part of it.

Another related set of hypotheses which has been formulated recently deal with the role of computation and computability in physics. Tipler (Ref. 16; cf. also my "law of increase of Q-logical depth," Ref. 9) has derived physical, in principle testable consequences from the postulate that the amount of information that can be processed in the lifetime of the universe be unbounded. Several authors have proposed (e.g., Ref. 10), for various reasons, that the laws of physics must be "computable." And I have proposed[9] that the Church–Turing hypothesis[17] be reformulated as the following physical principle:

> *Every physical system can be perfectly simulated by a universal model computing machine operating by finite means.*

In a broad sense what links all these ideas is that they attempt to derive physical consequences from postulates outside physics, specifically from epistemological assumptions, preferably ones we were "going to make anyway." If this turns out to be successful as a research program, another way of looking at it would be that epistemology is now being incorporated into physics, just as the scope of physics has historically been successively extended to take over other sciences such as astronomy, chemistry, geometry, and most recently computer science.

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