Determining the ratio $\frac{r_p}{r_e}$ for the outer planets.

In Figure 1 we consider the triangular configuration of the sun, earth, and a planet at a particular moment in time, t_1 in a heliocentric model. The distance to the earth from the sun is r_e and the distance from the sun to the planet is r_p. Suppose at some time in the year the earth the planet and the sun are in conjunction. Imagine this line SEP is extended to the Celestial Sphere, indicated by a star S_1. At conjunction define $t_0 = 0$, and at some later time, t_1 the earth has moved to a new position making an angle with respect to the direction to S_1 by an angle α

$$\alpha = 360^\circ \frac{t_1 - t_0}{T_e} \tag{1}$$

At the same time the planet has moved through an angle β

$$\beta = 360^\circ \frac{t_1 - t_0}{T_p} \tag{2}$$

Here T_e and T_p are the orbital periods of the earth and the planet. At the vertex of the sun we determine the angle γ

$$\gamma = \alpha - \beta \tag{3}$$

Now at time t_1 we make an observation of the angle, θ, of the planet with respect to the direction of S_1. We can now determine the angle ϕ at the vertex of the earth

$$\phi = 180 - \alpha - \theta \tag{4}$$

Hence, we know from our studies of Euclidian geometery that angle-side-angle information is enough to determine the other parameters of the triangle ΔSEP. In particular this will give us the ratio $\frac{r_p}{r_e}$. A plane figure drawing with a protractor and ruler would be one way to do this. Another way is to do a calculation, for example. In figure 1 we draw in a perpendicular line h from the vertex e to the r_p line. The angle δ at the vertex p is

$$\delta = 180 - \gamma - \phi. \tag{5}$$

$$h = s tan(\delta) \tag{6a}$$

$$h = (r_p - s) tan(\gamma) \tag{6b}$$

$$s = \frac{r_p tan(\gamma)}{tan(\delta) + tan(\gamma)} \tag{6c}$$

We combine equations 6a and 6b to obtatin a statement for s in equation 6c. From the vertex at s we recognize that $cos(\gamma) = \frac{r_p - s}{r_e}$ We can eliminate s from this equation with equation 6c. The result is

$$\frac{r_p}{r_e} = (\frac{tan(\delta) + tan(\gamma)}{tan(\delta)}) cos(\gamma) \tag{7}$$

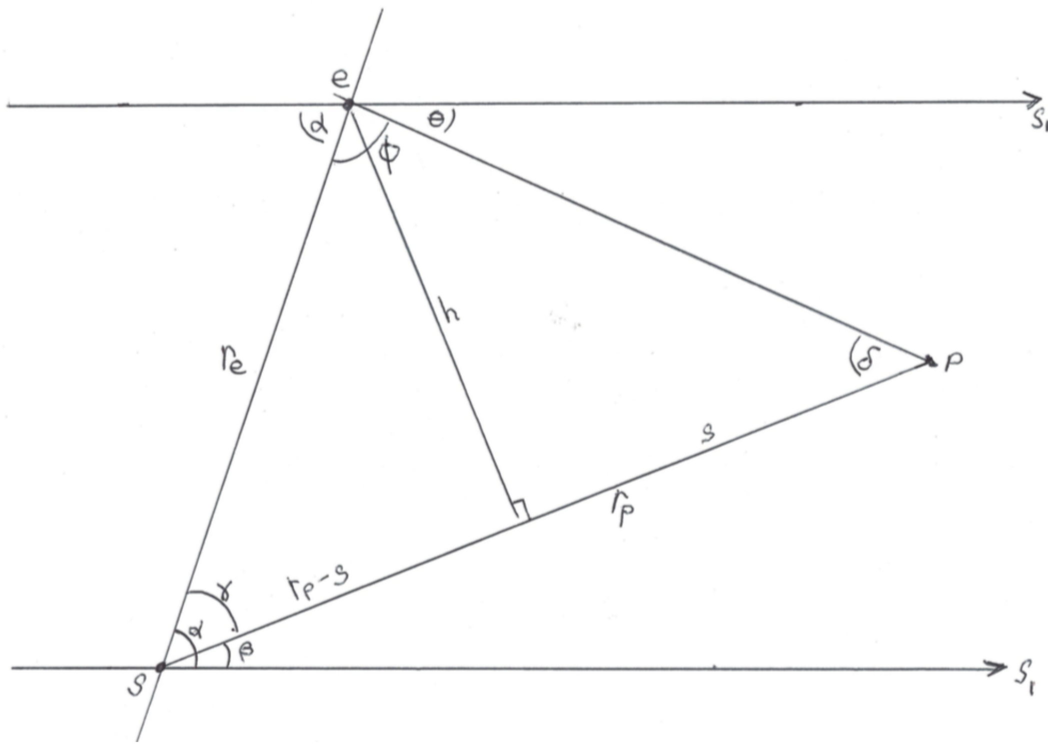

Figure 1: Triangular configuration of the sun and earth and a planet, ΔSEP at time t_1.