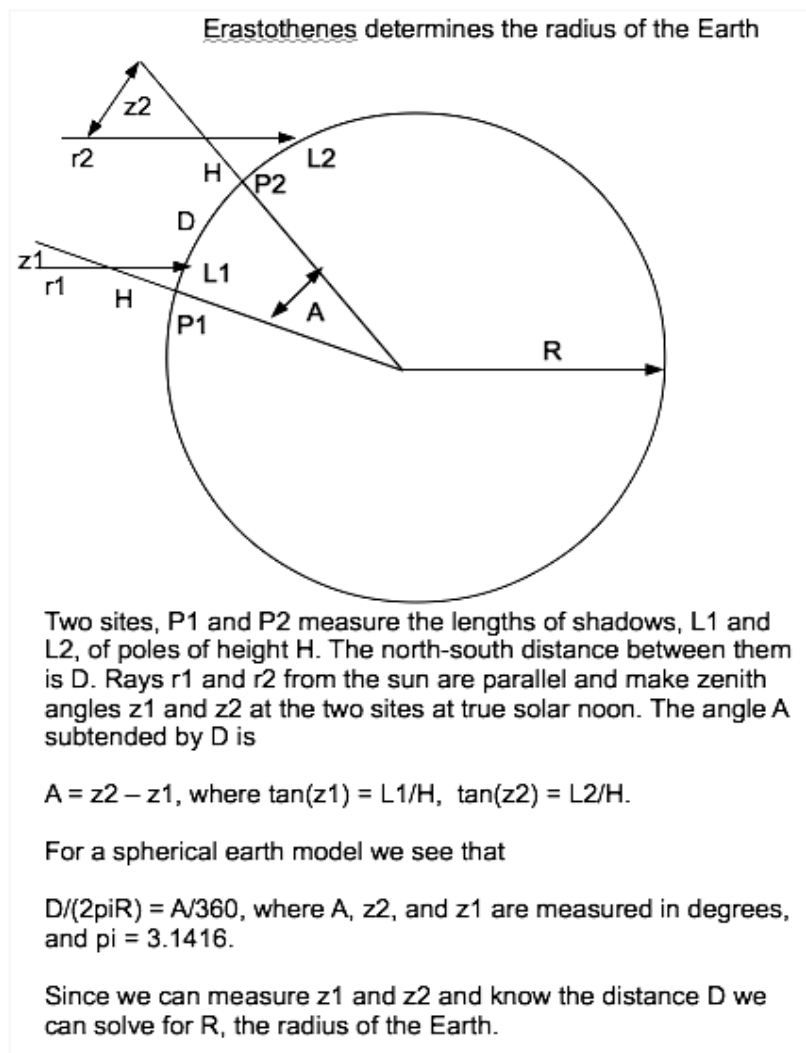# Astronomy 360 – Field Report

A Measurement of the Radius of the Earth

This measurement will rely on an ancient technique for determining the size of the Earth from the measurement of the lengths of shadows. It will be conducted by your chosen group of students. Ideally there will be two measurements by your group on the same day at the same time. The groups should be at least 100 miles apart on a true north-south line. This ideal situation is rarely possible. Instead of having such widely separated teams the instructor may provide the class with shadow measurements from another source for the day and time.

Erastothenes determines the radius of the Earth

Two sites, P1 and P2 measure the lengths of shadows, L1 and L2, of poles of height H. The north-south distance between them is D. Rays r1 and r2 from the sun are parallel and make zenith angles z1 and z2 at the two sites at true solar noon. The angle A subtended by D is

A = z2 – z1, where tan(z1) = L1/H, tan(z2) = L2/H.

For a spherical earth model we see that

D/(2piR) = A/360, where A, z2, and z1 are measured in degrees, and pi = 3.1416.

Since we can measure z1 and z2 and know the distance D we can solve for R, the radius of the Earth.
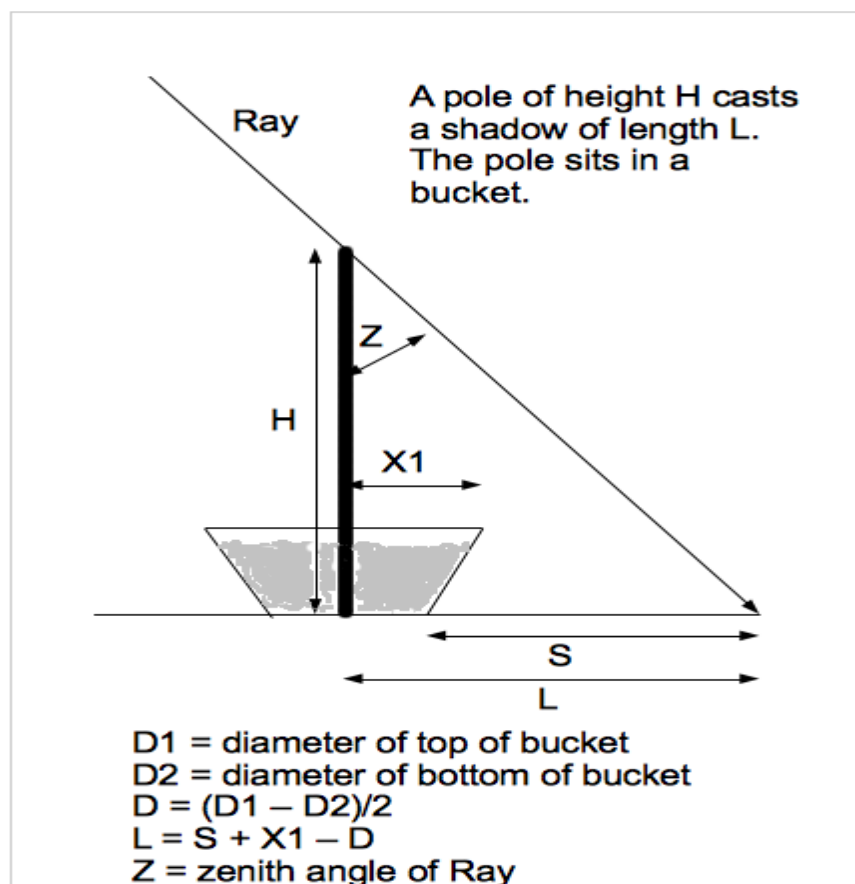
A proposed method of reliably measuring shadows is included in the ensuing discussion. If you find a better method explain what you did in your report. There is preparation your group must make before measuring the data. Attention to detail and care in the measurements will ensure that the time you devoted to the measurements was well spent. It is advised that your group practice making the measurement before the day selected for the entire class. This way your partners will agree on the procedures.

The measurements must be made during the solar meridian crossing. This time is known as true solar noon. You should start making measurements at least ½ hour before true solar noon and continue making measurements for at least ½ hour after true solar noon. You will know that you have captured true solar noon when the shadow lengths reach their minimum value. The shadows start becoming shorter until true solar noon. They start to become longer again after true solar noon. True solar noon can be estimated before hand by looking up sun rise and sun set times for the day in question. Solar noon occurs exactly half way between sun rise and sun set. **True solar noon is not civil noon!**

Procedures:
1) There are in principle two sets of teams designated as the "near" team at site P1 and the "far" team at site P2. Ideally sites P1 and P2 are separated along a north-south line, i.e., at the same longitude. In our case this is not usually practical. However, given the limited accuracy of the shadow measurements the longitudes of the two sites need not be identical. We want at least 100 miles between the two sites along a north-south line.
2) You must make your measurements around true solar noon. Be sure you get to your measurement site with enough time to set up your apparatus.
3) Find a reasonably level spot by placing the carpenter's level on the ground. The pole and ground should be at right angles. Use the carpenter's level to make the pole vertical.
4) One person should hold the pole vertically while the other person pours the bag of sand into the bucket.
5) If the sandbag is newly opened you will probably need to dampen it slightly. Don't pour in too much water or the sand will become a slush.
6) Once the sand bag has been emptied use the carpenter's level against the pole to make sure the pole did not tilt. Check the verticality of the pole on two sides that are perpendicular to each other.

7) Keep careful track of the quantities in the table below. Check the pole's verticality by gently placing the level against it periodically. This may be necessary if it is a windy day. You should see the shadow shortening towards noon and increasing in length after noon. If you are measuring the shadow on concrete or asphalt place masking tape at the terminus of the shadow and mark the tape where the shadow ends. If your are measuring the shadow on soil drive a nail into the ground where the shadow ends. The shadow will appear blurred because of the finite size of the sun. Be consistent in what you consider to be the shadow's end. You may need to discuss this with your partners.

8) The zenith angle at your site is called Z. TAN(Z) = L/H. L is the true length of the shadow and H is the height of the pole. Your calculator should give you the inverse tangent function, often designated as TAN^{-1} or ATAN on the calculator keys. Use the degree mode of the calculator rather than the radian mode for angles.

Ray

A pole of height H casts a shadow of length L. The pole sits in a bucket.

Z

H

X1

S

L

D1 = diameter of top of bucket
D2 = diameter of bottom of bucket
D = (D1 − D2)/2
L = S + X1 − D
Z = zenith angle of Ray

Cindy Rodriguez

Example Data Table

Date: 4/10/21

Location: culver city

bucket D1: 6.5

bucket D2: 3

D: 1.75

H: 12in

smallest z1:

smallest z2:

A = z2 – z1

R = 180*D/(A*pi)

Time	S	X1	L=S+X1-D	tan(z)=L/H	z = ATAN(L/H)
12:05 pm	6.5	3.5	8.25	0.68	34.50
12:35	7.5	3.5	9.25	0.77	37.62
1:05	8.5	3.5	10.25	0.85	40.50
1:35	9	3.5	10.75	0.89	41.85
2:05	9.5	3.5	11.25	0.93	43.15
2:35	10	3.5	11.75	0.97	44.39
3:05	10.5	3.5	12.25	1.02	45.59
3:35	11.5	3.5	13.25	1.10	47.83
4:05	12.5	3.5	14.25	1.18	49.89
4:20	13	3.5	14.75	1.22	50.86

This table also works well in a spread sheet.

Analysis

1) Fill in the data table with your measured zenith angles, z.

2) This is your P1 contribution to the class's data set. Send your data table as an email attachment to the instructor. Include the names of everyone who assisted in the measurement. He/she will post your results on the ASTR360 Lecture Notes page.

3) If a group travels far enough away from CSULA on the shadow measurement day their data can be used as the P2 site. The distant site must be at least 100 miles away on a true north south line. This is the distance on a fixed longitude between latitudes. Going even 1000 miles east-west at the same latitude is not useable.

4) The instructor will supply the zenith angles at Sacramento as a P2 site. No site in Southern California is a valid P2 site. The true N-S distance from the Southern California sites will be listed on the web link with the USNO zenith angle predictions.

5) The P1 sites are your class mates results. **Look at every data set that was submitted. You judge whether or not the data are reliable.** State your criteria. You can not use the known radius of the earth as a criterion for selecting a data set.

6) Reliable data will show the shadow lengths decreasing until solar noon and then increasing after solar noon.

7) Find the radius of the earth for each reliable pair of P1-P2 and take the average to report the radius of the earth. This is the best determination from our data sets.

8) Everyone must turn in a report doing the analysis even if you were not able to assist in the data measurement. Include a short narrative in the report describing the experimental difficulties, if any, in making the measurements. Can you suggest improvements in procedures.

9) Make a plot of a good data set you choose showing the zenith angle on the vertical axis and the time of day on the horizontal axis. A spread sheet plot is acceptable.

10) Everyone whose name appears on the posted data tables submitted for the whole class's use will receive at least 10 points out of the 20 points assigned to the report.

11) For 25% extra credit use the data to determine the height of the sun above a flat earth model. In order to get full credit for this the answer must be right. Use any resource you can find to determine the height. There will not be partial credit for extra credit. The answer is either right or wrong.

$$\tan(z1) = L1/h \qquad \text{and} \qquad \tan(z2) = L2/h \quad .$$

Diagram deduced from the discussion in the Book of Chen Zi from the Zhou bi suan jing which describes how the height of the sun, s, above a flat earth can be found. This is also the height of the sky above the Earth. It is note worthy that the same information about shadow lengths can lead to very different conclusions depending upon the model we assume.