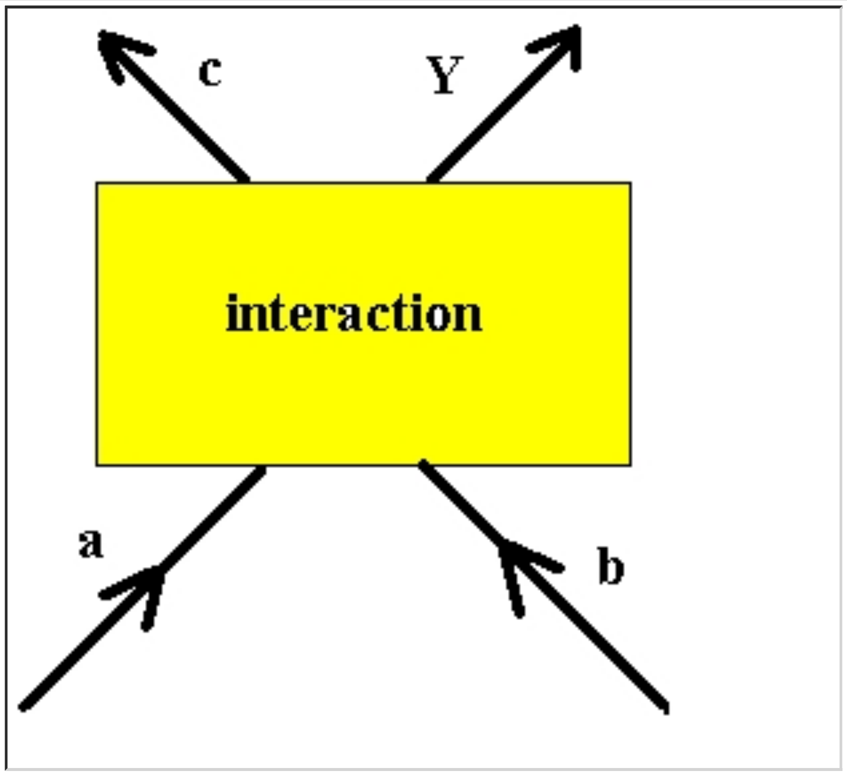
Lecture 1 - Kinematics

Consider the reaction a+b -> c +Y. The picture below depicts this process with the time axis running vertically. Whatever the interactions are in the interaction region, we know that the conservation laws of momentum and energy produce relationships between the energies and momenta.



The 4-momenta are

$$p_a^\mu = (E_a, \vec{p}_a),\ p_b^\mu = (E_b, \vec{p}_b)$$

$$p_c^\mu = (E_c, \vec{p}_c),\ p_Y^\mu = (E_Y, \vec{p}_Y)$$

Conservation of 4 momenta means

$$p_a^\mu + p_b^\mu = p_c^\mu + p_Y^\mu, \text{ or}$$

$$\vec{p}_a + \vec{p}_b = \vec{p}_c + \vec{p}_Y, \text{ and } E_a + E_b = E_c + E_Y$$

and

$$E = \sqrt{p^2 + m^2}$$

We will suppose that particle a is the projectile and b is the target. If we measure the 4-momentum of outgoing particle c, then we know what the 4-momentum of Y is using the energy-momentum conservation laws. We define new variables to indicate the 4 momentum transfer in the process.

$$p_a^\mu - p_c^\mu = p_Y^\mu - p_b^\mu, \text{ let } q^\mu = (\omega, \vec{q})$$

with

$$\omega = E_a - E_c, \text{ and, } \quad \vec{q} = \vec{p}_a - \vec{p}_c$$

Elastic Scattering

Consider the case of elastic scattering, a = c, and the target b is stationary, then,

$$\omega = E_Y - m_b, \text{ or}$$

$$\omega = \sqrt{m_b^2 + q^2} - m_b, \text{ where}$$

$$q^2 = \vec{q} \bullet \vec{q}, \text{ thus}$$

$$\omega + m_b = \sqrt{m_b^2 + q^2}, \text{ squaring we get,}$$

$$m_b^2 + \omega^2 + 2m_b\omega = m_b^2 + q^2$$

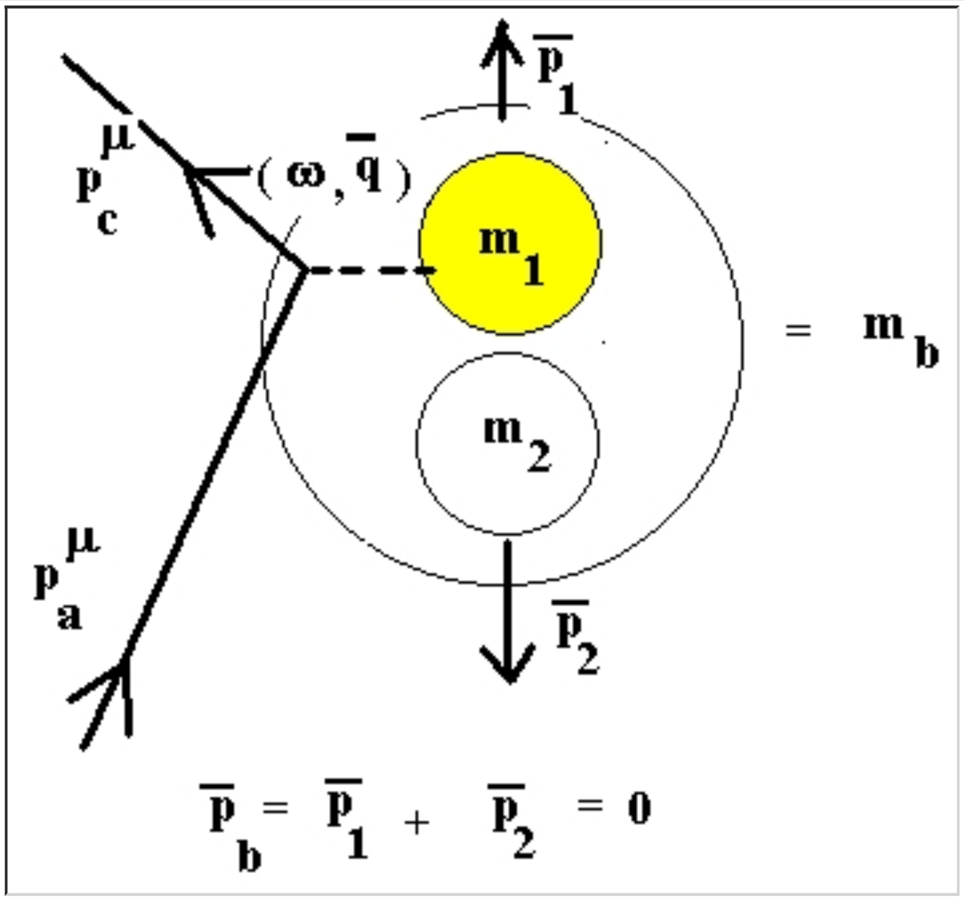
$$2m_b\omega = q^2 - \omega^2 = -q_\mu q^\mu \equiv Q^2,$$

so for elastic scattering we note that

$$\frac{Q^2}{2m_b\omega} = 1.$$

Scattering from Constituents

Consider this case then. We imagine that the target consists of two pieces, that is, it has substructure. The incident particle, a, scatters off component 1, where we assume that the whole 4 momentum lost by a is transferred to particle 1.



$$m_b = \sqrt{m_2^2 + p_1^2} + \sqrt{m_1^2 + p_1^2} + \Delta(p_1)$$

$\Delta(0)$ **is the reaction Q - value when** $p_1 = 0$**. The final state Y consists of** m_1 **with momentum** $\vec{p}_1 + \vec{q}$**, and** m_2 **with momentum** $-\vec{p}_1$ **so that the total momentum of Y is** $p_Y = \vec{p}_1 + \vec{q} - \vec{p}_1 = \vec{q}$**. The energy of Y is**

$$E_Y = \sqrt{m_2^2 + p_1^2} + \sqrt{m_1^2 + (\vec{p}_1 + \vec{q})^2}$$

thus , $(\omega, \vec{q}) = (E_Y - m_b, p_Y)$

We can expand the equation for the energy transfer to arrive at

$Q^2 \equiv q^2 - \omega^2$, **and**

$$\frac{Q^2}{2m_1\omega} = \sqrt{1 + \frac{p_1^2}{m_1^2}} - \frac{\vec{q}\cdot\vec{p}_1}{m_1\omega} - \frac{\Delta^2(p_1)}{2m_1\omega} + \frac{\Delta(p_1)}{\omega}\sqrt{1 + \frac{(\vec{p}_1 + \vec{q})^2}{m_1^2}},$$

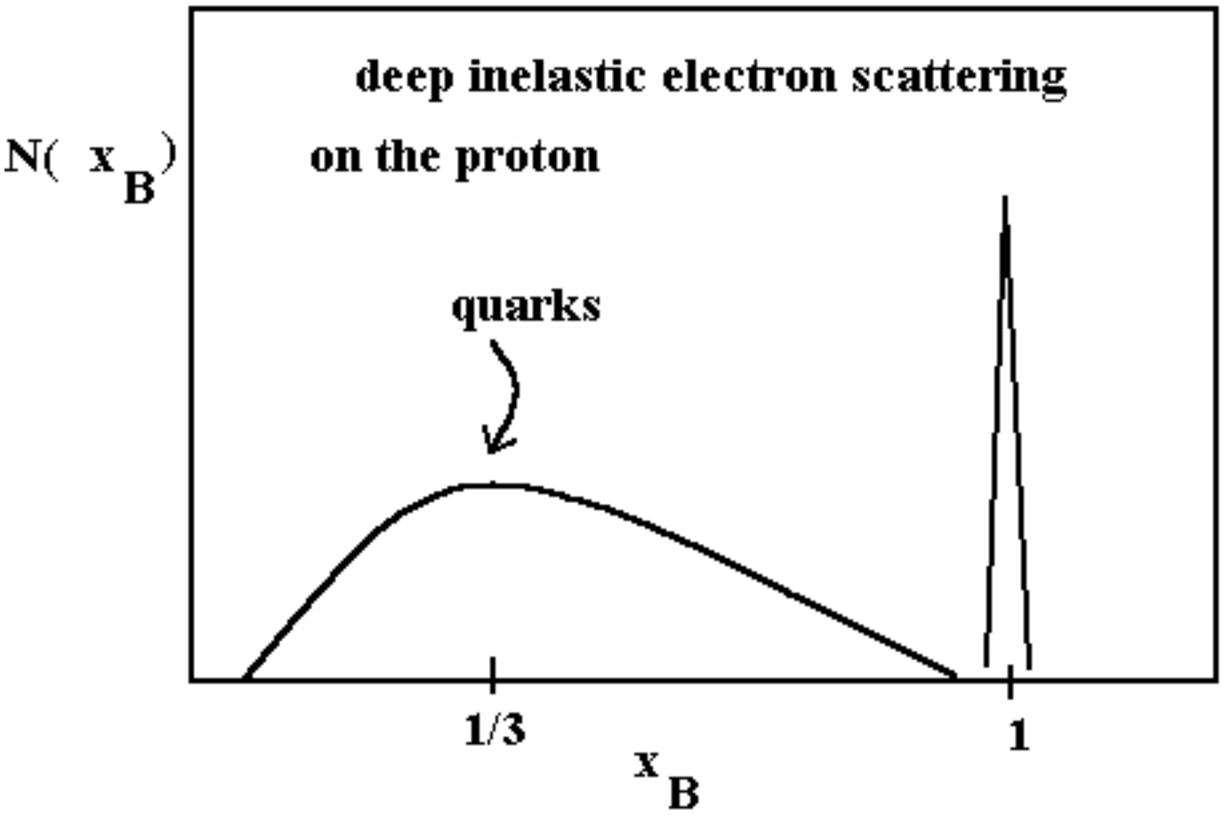
call $x_B = \dfrac{Q^2}{2m_p\omega}$ **the Bjorken x value where** m_p **is the mass of the proton. For elastic scattering on the proton** $x_B = 1$**.**

$$\text{As } p_1 \to 0,\ x = \frac{Q^2}{2m_1\omega} = 1 + \frac{\Delta(0)}{\omega}\left(1 - \frac{\Delta(0)}{2m_1}\right), \text{ and } \Delta(0) < 0.$$

The condition where x is approximately 1 is called quasi-elastic scattering. In this kinematical condition particle a loses energy and suffers a 3-momentum change consistent with scattering off an object with the mass of constituent 1. In the case of electrons scattering off a nucleus a broad peak appears at approximately x = 1 when we set mass 1 equal to the proton mass(then the x value is the Bjorken x).

If we only measure the scattered electron energy, then the measurement is called an inclusive experiment. These are also called "single-arm" measurements. If we measure the scattered electron in coincidence with another particle, say mass 1, then we call the measurement "exclusive". Exclusive experiments are necessarily coincidence experiments.

If the spectrum of high energy electrons scattered off the proton is measured and we plot the number scattered as a function of the Bjorken x variable we obtain a result similar to that seen below. A broad peak appears at x-Bjorken of about 1/3. If we interpret this bump as due to elastic scattering then we would have to assume that the electrons are elastically scattering off constituents of the proton of mass about 1/3 the proton mass. That is, if we had used 1/3 the proton mass for the mass 1 constituent then the bump would appear at x = 1. The quasi-elastic scattering on the quarks produces a broad spectrum for the same reason that the quasi-elastic scattering on protons in the nucleus produces a broad spectrum. Namely, the constituents receiving the 4-momentum transfer are in motion.



So generally speaking if we plot the scattered electron energy, for any sort of target we can conclude that the different values of the Bjorken x correspond to different masses of the constituents absorbing the transferred 4-momentum:

$x_B < 1$ means scattering on a constituent less massive than the proton

$x_B = 1$ means scattering on a constituent about as massive as the proton

$x_B > 1$ means scattering on a constituent more massive than the proton, such as a correlated pair of nucleons.

If we see a peak in the spectrum of $N(x_B)$ vs x_B at a location x'_B , then a kinematical interpretation of the peak is that electrons are scattering off a constituent with a mass $m_1 = x'_B \cdot m_p$.

Missing momentum and missing energy

In an exclusive reaction (a coincidence measurement such as A(e,e'p)B) we measure the energy of the scattered electron and this tells us the 4-momentum absorbed by the target. In addition we also measure the 4-momentum of one of the particles ejected from the target. The missing momentum is simply the momentum we did not explicitly measure. We know what it is, of course, because we have conservation of 4-momentum. The same argument applies to the missing energy.

Measuring the outgoing electron's energy we determine the 4-momentum transferred.

$$q^{\mu} = (\omega, \vec{q}).$$

Measuring the 4-momentum of the knocked out proton, $(E_p, \vec{p}_p)$ we deduce that the missing 4-momentum is

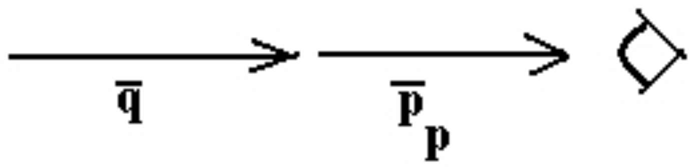
$$p_x^{\mu} = (\omega - E_p, \vec{q} - \vec{p}_p)$$

In the simple minded picture we have of scattering from a single constituent (the impulse approximation) the missing momentum of the undetected fragment (x = 2 in our diagram above) is the negative of the momentum our detected proton had in the nucleus before it absorbed the transferred 4-momentum from the electron.

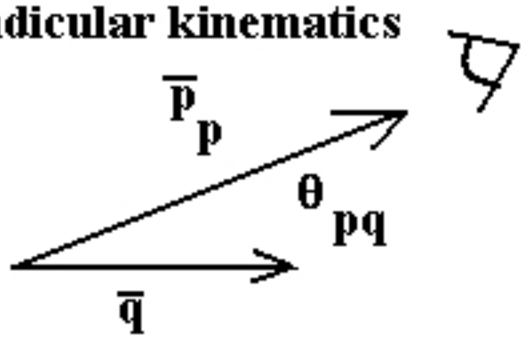
Parallel and Perpendicular Kinematics

If we detect the proton knocked out of the nucleus along the direction of the 3-momentum transfer, we refer to this as parallel kinematics. The definition of perpendicular kinematics is not so sharply made. Basically, perpendicular kinematics is not parallel kinematics. It is not required that the proton be detected at 90 degrees to the three momentum transfer. In parallel kinematics we can separate the longitudinal and transverse resopnse functions. In perpendicular kinematics we determine the longitudinal-transverse interference response function.

parallel kinematics



perpendicular kinematics



θ_{pq} is normally large, but not necessarily 90 degres