Thermal Analysis of the Proposed C200 Calorimeter

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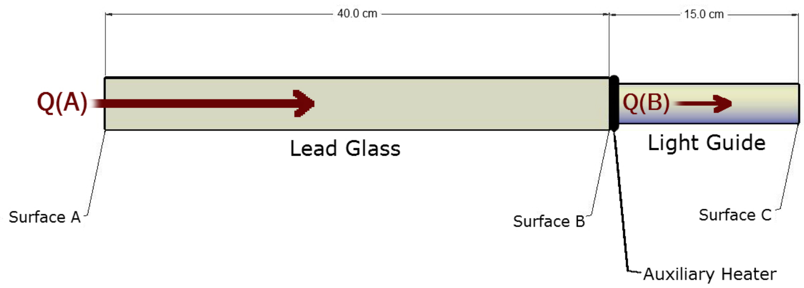
Abstract

Thorough analysis of the C200 calorimeter's thermodynamic properties is critical for understanding its ability to produce desired outcomes and for determining its design feasibility. Several components of the calorimeter must be carefully assessed in order to accomplish such an analysis. Its current design calls for an array of blocks of lead glass attached to glass cylinders which act as light guides for PMTs. The purpose of this calorimeter design is to maintain transparency in the lead glass by affixing a heater that allows for continuous annealing, which eliminates the known effects of reduced transparency due to radiation. Furthermore, an auxiliary heater is attached to each light guide in order to achieve and maintain an effective longitudinal temperature gradient throughout the array. The entire arrangement of heaters, lead glass, and light guides is insulated by panels of foam glass on all sides of the calorimeter to reduce heat loss in the system. The purpose of analysis is to determine how the proposed calorimeter parameters may attain the most effective temperature gradient and how the C200 design performs during heating and cooling intervals.

1 Primary Heater Investigation

1.1 Required Heat

The first step in analyzing heat transfer within the calorimeter is to determine heat flux in each lead-glass-and-light-guide block. Because the block's components are glued, we can assume that thermal conductance provides the most pertinent transfer of heat. For purposes of simplification, we first ideally analyze the system without any transverse heat loss, and we assume symmetry in each block assembly. By doing this, the path of heat flux can be interpreted as one-dimensional along the length of each block.



Above, diagram depicts a single lead glass and light guide block. Q_A is the heat provided by the main heater and Q_B is the heat provided by the auxiliary heater.

From Fourier's law, we find the following values for necessary heat in a single block to achieve the desired temperature gradient:

$$Q_i = \frac{k_i A_i \Delta T_i}{s_i}$$

$$Surface\ A \rightarrow 225\ °C$$

$$Surface\ B \rightarrow 175\ °C$$

$$Surface\ C \rightarrow 50\ °C$$

$$Q_A = Q_{Lead\ Glass} = 0.16\ W$$

$$Q_{Light\ Guide} = 0.62\ W$$

$$Q_B = Q_{Light\ Guide} - Q_{Lead\ Glass} = 0.46\ W$$

With this information, we determine that the main heater must provide 0.16 Watts per block and the auxiliary heater must provide an additional 0.46 Watts per light guide. Since the design calls for 400 lead glass blocks with 200 light guides, we find that the total heat necessary to achieve our desired temperature gradient is

$$\sum Q_A = 64\ W$$

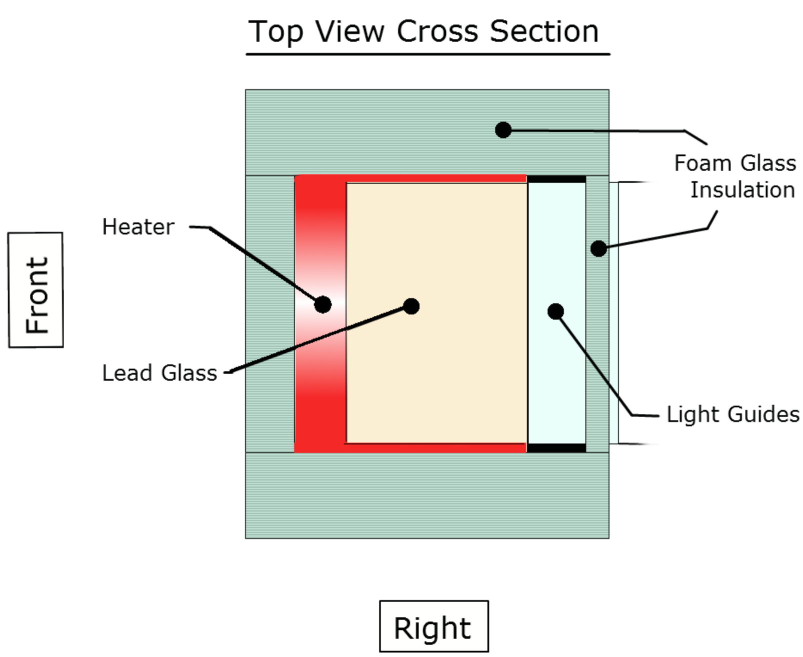
$$\sum Q_B = 92\ W$$

$$Q_{total} = \sum Q_A + \sum Q_B = 156\ W$$

While the previous results and calculations provide an ideal estimation of heat flux within the calorimeter, they disregard inevitable heat loss through all sides of the insulated system. Foam glass, the material used for insulation, has very low thermal conductivity ($k = 0.05\ W/m \cdot K$) and is thus a good insulator. Nonetheless, heat loss is significant through the foam glass panels due to the high temperature required for the calorimeter. Most notably, we can predict the existence of a transverse temperature gradient through the blocks which will require additional side heating to maintain an appropriate longitudinal temperature gradient throughout the calorimeter.

1.2 Side Heaters

Due to the magnitude of transverse heat loss along the longitudinal faces of the lead glass blocks, we determine that heating on the sides is also necessary in order to attain a one-dimensional temperature gradient throughout the lead glass. Since we can easily find the potential heat loss through the insulation given a certain inside and outside surface temperature, we can also estimate the configuration and composition of the side heaters.

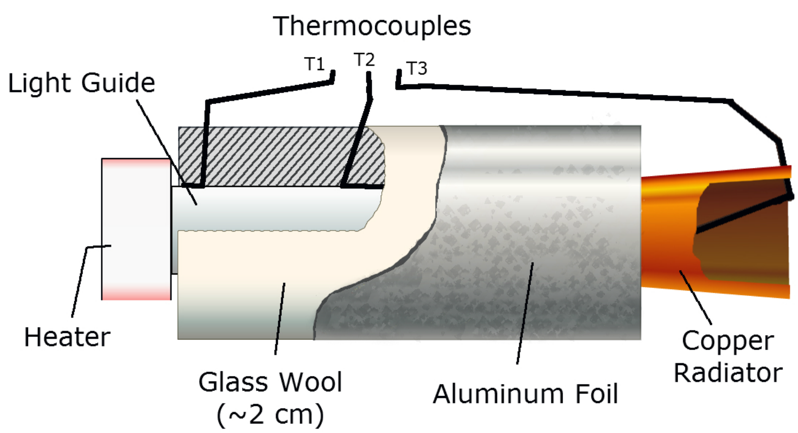


Cross-sectional top view depicting heaters, lead glass, light guides and insulation as arranged in the C200 calorimeter

Assuming that we attain our desired linear temperature gradient in the lead glass, the temperature at a given longitudinal coordinate x is defined by the linear expression $T(x) = 225 - 125x\ [^{\circ}C]$. Furthermore, we calculate the heat loss through the insulating panels and along the same longitudinal coordinate x as $Q(x) = \frac{k_f L x}{s_f}[T(x) - T_\infty]$. With known values, we calculate that we require $Q = 12\ W$ per side and $48\ W$ total for side heating. Of course, this estimation assumes very ideal circumstances and does not accurately predict the convective heat loss effect from surrounding air and through insulation joints, nor the conductive heat loss from steel bracing in the calorimeter. With knowledge of a more accurate expression, we can balance the potential transverse heat loss to find the heat required to maintain linearity in the longitudinal temperature gradient.

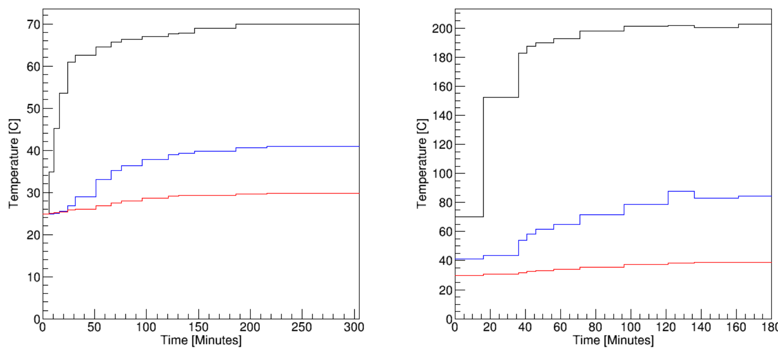
2 Study of Light Guide Heat Transfer

Next, it is necessary to fully understand the temperature gradient within a block. An experiment was thus setup in order to determine whether the gradient was linearly distributed along the length of the light guide. With a heat source attached to one end of an insulated light guide, and three thermocouples attached along its length, the experiment was able to determine the degree of linearity of the temperature gradient. Additionally, a heat exchanger made of copper ribbons was attached near the unheated end of the light guide in order to simultaneously determine its cooling efficiency.



The copper radiator acts as a heat exchanger to ensure and maintain appropriate temperature at the light guide's unheated end.

For this experiment, temperatures were measured from the three thermocouples at varying time intervals while the setup heated to $70\,^\circ C$ and until it reached an equilibrium temperature throughout. Once this data was collected, the experiment was repeated until a final equilibrium temperature of $200\,^\circ C$ was reached. After data was recorded, it was observed that the measured temperatures at the three thermocouples did not apparently show a linear relationship with respect to each other.



Two separate studies showing profiles of temperatures recorded by each of three thermocouples. Both profiles display the efficacy of a simple copper radiator in reducing the light guide's temperature

Our most important finding the light guide experiment was the effectiveness of the copper radiator. Even when the light guide's heated end approached an equilibrium temperature of 200 $^{\circ}C$, its unheated end remained below 40 $^{\circ}C$. Since our goal is to constrain the temperature at the light-guide-PMT interface to 50 $^{\circ}C$, and plan on ensuring this temperature restriction with forced convective cooling, we are very pleased to observe that the simple copper radiator can achieve this result.

3 Heat Transfer Analysis for Lead Glass

Moving forward with analysis, we examine heat transfer in the calorimeter's lead glass blocks. Assuming that we can attain a reasonable linear temperature gradient through sufficient side heating, we are interested in studying the transient process of heating the calorimeter.

3.1 Heating Period

For purposes of this analysis, we suppose that sufficient heat flux maintains a constant temperature at the front and back faces of lead glass blocks. Such a supposition depicts each of the two boundary surfaces exposed to a heat reservoir, and is made to reduce the boundary conditions of the heating process to constants. Now utilizing the heat equation for transient heat transfer, we quickly find that the solution is non-trivial. In order to solve the equation analytically, we must amalgamate several methods for solving boundary-value PDEs.

With $u,\ x,\ t$, and α representing temperature, longitudinal coordinate (as this is a one-dimensional heating problem), time, and diffusivity ($k/\rho{\cdot}C_p$) respectively:

$$\alpha \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$$

Boundary Conditions:

$u(x,0) = T_\infty$
Ambient temperature

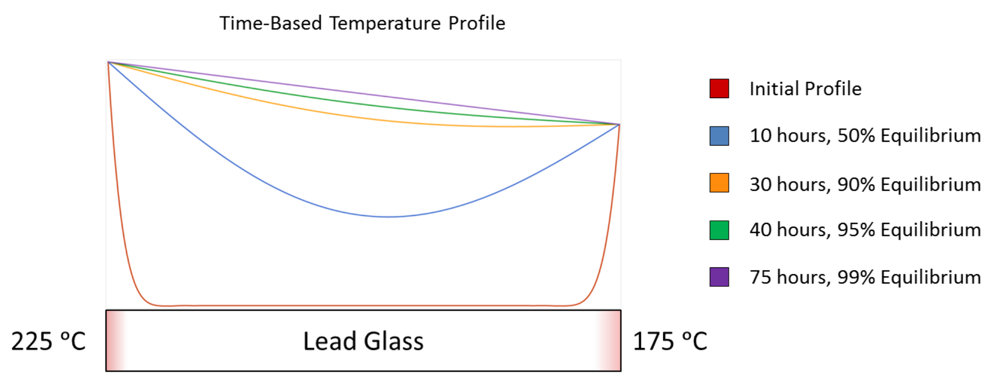
$u(0,t) = T_1$
Constant temperature at one boundary surface

$u(L,t) = T_2$
Constant temperature at other boundary surface

With known boundary conditions, our result is an approximate solution accurate to 5 $^\circ C$ for $t > 0$ and verified with numerical analysis.

$$u(x,t) = \left[T_1 + \left(\frac{T_2 - T_1 - T_\infty(L-x)}{L}x\right)\right] - \sum_{k=1}^{\infty} \frac{2}{k\pi}\left[T_2 - T_2\cos(k\pi)\right]\sin\left(\frac{k\pi x}{L}\right)\cdot e^{-(\frac{k\pi}{L})^2\alpha t}$$

With this solution, we are able to better understand and visualize the effect of boundary parameters and heating time.

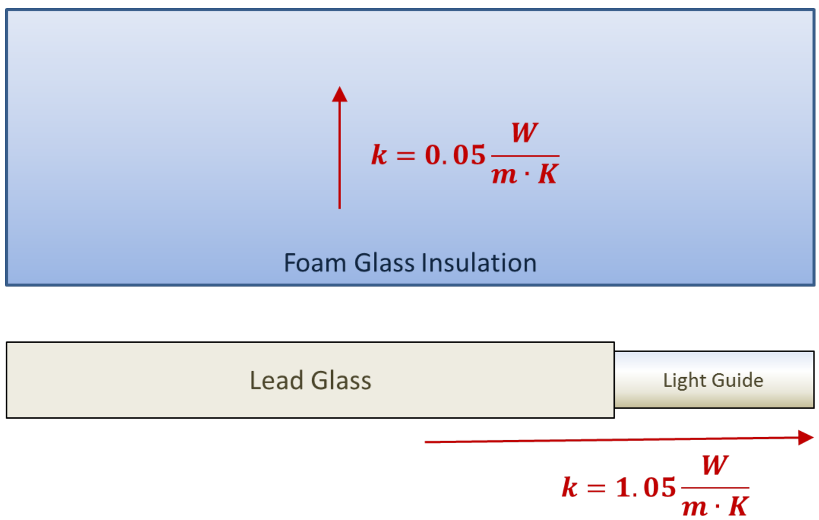


Transient temperature profile for temperature vs. longitudinal coordinate.

These calculations allow us to make estimations pertinent to experimental logistics. For example, if we find that the calorimeter may perform adequately with 90% temperature equilibrium in the lead glass blocks, then we may save at least 35 hours of experimental set-up time.

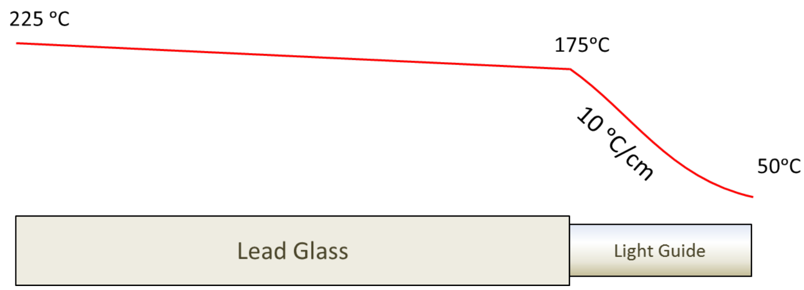
3.2 Cooling Period

Transient heat transfer analysis in the lead glass blocks is rather complex due to the conductive, convective, and radiative cooling parameters within the insulated system. Nevertheless, we can reasonably predict that a majority of the heat contained within the lead glass blocks will transfer through the light guides. There is no forced convection from the air contained within the calorimeter's insulation; both air and insulating foam glass have significantly low thermal conductance relative to that of the lead glass blocks and light guides; and the back end of the light guides are exposed to forced convection. Thus, we can justifiably expect efficient cooling in the case of immediate thermal regime modification.



Lead glass blocks and light guides conduct heat roughly 20 times better than foam glass and roughly 35 times better than air ($k_{air} \approx 0.03 \frac{W}{m \times K}$)

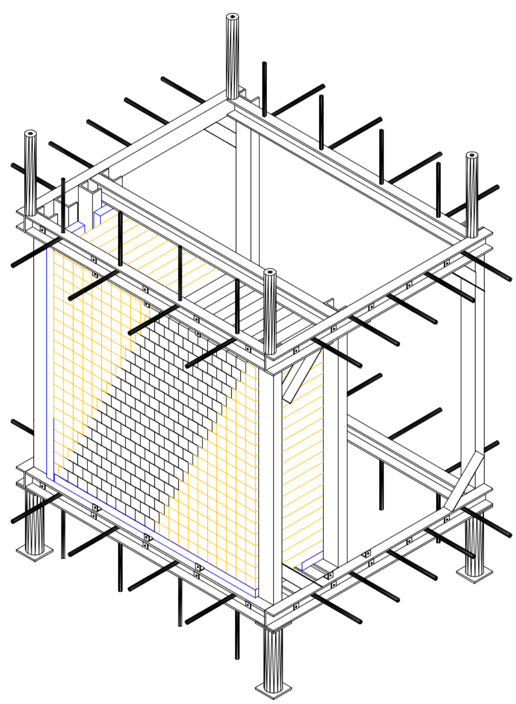
Furthermore, we anticipate a maximum longitudinal temperature gradient of 10 $°C/cm$ – which is within the acceptable range for glass – in the light guide region. This gradient is expected at the very onset of cooling as the heaters are off and the intermediary surface between lead glass and light guides has not yet begun to cool down, meanwhile rapid cooling by forced convection increases the magnitude of the temperature gradient.



Lead glass, light guide, and predicted temperature gradient.

4 Expansion Under Modified Thermal Regime

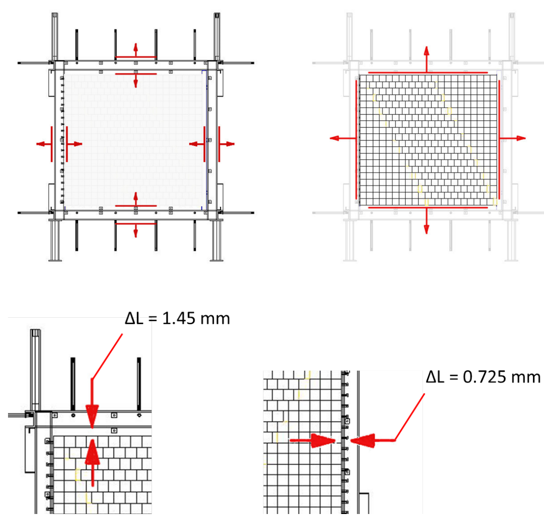
Due to the substantial temperatures used in the proposed thermal regime, an investigation must be made as to whether thermal expansion of the calorimeter components can compromise mechanical stability. In its design, the calorimeter is supported and retained by steel bracing.



3D isometric model of C200 calorimeter and steel bracing by Galust Sargsyan

Steel has a significantly higher coefficient of expansion than lead glass (13×10^{-6} vs. $6 \times 10^{-}6\ m/m \cdot K$); we can therefore expect greater expansion in the bracing than in the lead glass blocks. Furthermore, steel's thermal conductivity is roughly 30 times that of lead glass ($24\ vs.\ 0.8\ W/m \cdot K$), and we therefore anticipate its temperature to rise more quickly while heating and drop more quickly while cooling. Consequently, we deduce that the steel bracing will experience greater expansion more rapidly than the calorimeter itself. In order to retain the lead glass blocks in their relative positions and maintain the calorimeter's overall shape, the steel bracing utilizes spring bracing panels. These panels are situated along the perimeter of the calorimeter's sides and top, and are designed to provide a persistent compressive force through variable thermal regimes.

During the heating cycle, we suppose that thermal expansion will have its greatest effect as gaps form between the disproportionally expanding materials. Using known material and design properties, we calculate that the longitudinal and transverse expansion of the steel bracing combined with the vertical and horizontal expansion of the lead glass blocks will result in gaps up to 1.45 mm on the top and 0.725 mm on the sides. While these gaps are nominal, they will nonetheless be resolved with the aforementioned spring bracing panels.



While the calorimeter cools down, as in the case of immediate termination of heater operation, we are primarily concern with the behavior of the lead glass block array. Since we expect the peripheral blocks to cool slightly quicker, we anticipate the formation of gaps between blocks. While this effect is inevitable, we do not expect these gaps to exceed 10 microns in width. In addition, any compressive force applied to the lead glass blocks by the steel bracing due to the quicker contraction of the latter is again reconciled by the spring bracing panels. We can thus conclude that the calorimeter and it bracing will retain its mechanical stability during both heating and cooling cycles.

5 Lead Glass Annealing

The goal of the C200 calorimeter design is to maintain continuous heating in order to eliminate radiation damage in lead glass blocks. To better understand this process, we have undertaken an extensive study on the annealing process. The goal of the experiment was to study the relationship between annealing temperature, duration, and efficacy. Several lead glass blocks visibly affected by radiation damage were subjected to heat soaking for varied durations at various temperatures. Using a laser to measure transparency, we were able to study the absorption before and after annealing, and thus understand the factor, which we term DI, governing the rate of radiation damage reduction. DI is dependent on the base absorption of the lead glass block, which we discovered must be found experimentally and varies subject to the individual block. By extending the durations of annealing, we approached maximum damage reduction, and thus approached a value for base absorption, the logarithm of which we term $a1$. Furthermore, we observe and study the absorption due to radiation damage before any annealing has occurred; we term the logarithm of this term $a2$. Finally, we express a value R as the ratio of light transmitted through the lead glass block to light transmitted in air; the logarithm of R defines the absorption of the block. The derived relationship is as follows:

$$\ln(R) = -a1 - a2 \cdot e^{-DI}$$

The following table represents the data acquired for all of the annealing events involved in this study. It should be noted that outlying data has been highlighted in red and is a result of instrument uncertainty during the respective measurements. Although we observe varying results as a consequence of varying values of $a1$, we conclude that the heat annealing process is highly effective in eliminating radiation damage.

Block	Run Tim	t_{Eff} [Hrs]	Run Temper	Date	R_{Before}	R_{After}	$a_{Before} = \ln R_{Be}$	$a_{After} = \ln R_{Af}$	$a1 = \ln R_{Afte}$	$a2 = a_{Before}$-	DI	DI (Incremental)
		Effective annealing time			ratio of [current through glass]/[current through air]	ratio of [current through glass]/[current through air]	Log of absorption before	Log of absorption after	Log of base absorption	Log of radiation damage absorption	Total damage reduction rate as ΔI	Incremental damage reduction rate as ΔI
5	4	2.83	200	7/27/2014	0.046	0.481	3.079	0.732	0.576	2.503	2.78	2.78
	24	23.5	250	8/19/2014	0.046	0.520	3.079	0.653	0.576	2.503	3.48	0.70
	16	15.5	250	8/20/2014	0.046	0.561	3.079	0.578	0.576	2.503	7.25	6.54
	16	15.5	225	8/23/2014	0.046	0.562	3.079	0.576	0.576	2.503	-	-
6	2	0.83	200	7/28/2014	0.049	0.304	3.016	1.191	0.853	2.163	1.86	1.86
	24	23.5	250	8/19/2014	0.049	0.412	3.016	0.886	0.853	2.163	4.18	2.32
	16	15.5	250	8/21/2014	0.049	0.434	3.016	0.835	0.853	2.163	-	-
	16	15.5	225	8/23/2014	0.049	0.426	3.016	0.853	0.853	2.163	-	
7	2	0.75	250	7/29/2014	0.046	0.333	3.079	1.100	0.919	2.160	2.48	2.48
	12	11.5	250	8/13/2014	0.046	0.363	3.079	1.013	0.919	2.160	3.13	0.65
	16	15.5	250	8/20/2014	0.046	0.361	3.079	1.019	0.919	2.160	3.07	2.42
	16	15.5	225	8/23/2014	0.046	0.399	3.079	0.919	0.919	2.160	-	
8	4	2.92	250	8/3/2014	0.028	0.527	3.577	0.641	0.598	2.979	4.24	4.24
	12	11.5	225	8/11/2014	0.028	0.586	3.577	0.534	0.598	2.979	-	-
	16	15.5	250	8/20/2014	0.028	0.534	3.577	0.627	0.598	2.979	4.61	0.38
	16	15.5	225	8/23/2014	0.028	0.550	3.577	0.598	0.598	2.979	-	
9	4	2.75	225	8/6/2014	0.008	0.384	4.861	0.957	0.792	4.069	3.20	3.20
	12	11.5	250	8/13/2014	0.008	0.451	4.861	0.796	0.792	4.069	6.82	3.62
	16	15.5	250	8/21/2014	0.008	0.453	4.861	0.792	0.792	4.069	-	-
	16	15.5	225	8/23/2014	0.008	0.453	4.861	0.792	0.792	4.069	-	
10	8	7.41	225	8/8/2014	0.019	0.452	3.961	0.794	0.732	3.229	3.95	3.95
	16	15.5	250	8/10/2014	0.019	0.474	3.961	0.747	0.732	3.229	5.39	1.44
	12	11.5	225	8/11/2014	0.019	0.477	3.961	0.739	0.732	3.229	6.06	4.62
	16	15.5	225	8/23/2014	0.019	0.481	3.961	0.732	0.732	3.229	-	
11	2	0.75	225	8/11/2014	0.338	0.485	1.086	0.724	0.594	0.492	1.33	1.33
	16	15.5	250	8/21/2014	0.338	0.545	1.086	0.607	0.594	0.492	3.65	2.32
	16	15.5	225	8/23/2014	0.338	0.552	1.086	0.594	0.594	0.492	-	

6 Conclusion

In summary, we have undergone a thorough investigation of the thermal properties of the C200 calorimeter. Our investigation began with the heating elements, and we found an expression to dictate the required heat from our primary heater as well as from the side heaters. By satisfying this requirement, we will attain a one-dimensional temperature gradient in the lead glass blocks. Further analysis will be conducted to find a required heat magnitude which reflects realistic complexities in design elements; such analysis will allow us to better understand the transient heating process as well as the effect of convective cooling element parameters.

After a primary analysis was accomplished, we initiated a study of heat transfer within the light guides. This study ultimately provided us qualitative results reflecting the efficacy of a radiator as a cooling element. We were very pleased to find that the copper radiator alone – without forced convection – was effective in cooling the light guide. The experiment also allowed us to observe the transient heating process in an insulated cylinder. While the latter observation was only qualitative, it provided additional means to understand conductive heat transfer in a light guide.

Next, we analyzed heat transfer in the lead glass blocks. Beyond primary analysis, we were interested in understanding transient heat transfer in the C200's lead glass array. Our concern and goal in analysis was the time required to heat the array to an equilibrium temperature gradient. A thorough and arduous mathematical analysis proved necessary, but we ultimately came to derive an equation which reasonably depicts a temperature gradient at a given time. This approximation was utilized in finding the time required for the calorimeter to arrive at a desired thermal regime – a given percentage of equilibrium heating, for example. Likewise, we had an interest in understanding the heat transfer process during cooling of the calorimeter. This information is critical, as in the case of emergency shutdown or interruption of power. While further analysis is necessary to fully understand the posed scenario, we subjectively investigated the behavior of heat transfer under these conditions. Our conclusion, in this case, was that most of the heat will travel through the lead glass and light guides, as opposed to traveling through the insulation of the calorimeter. Since the light guides are subjected to forced convection cooling, we propose a quick cool-down relative to the duration of heating.

We then analyzed the overall thermal expansion of the design. Primarily, we wanted to understand whether the mechanical stability of the design may be compromised by heat expansion. Our results showed that the proposed C200 design will only experience a negligible magnitude of thermal expansion; furthermore, the expansion that occurs is fully mediated by spring bracing panels during both heating and cooling cycles.

After coming to an overall understanding of the thermodynamic and mechanical properties of the calorimeter, we were remiss without a physical analysis of the annealing effect which signifies the C200 calorimeter. Our analysis thus began with a general study of the annealing effect. This study involved annealing damaged lead glass for various durations at varied temperatures. With data from this study, we were able to interpolate and validate the dependence of both temperature and time on the effectiveness of annealing in reducing radiation damage. Our technique involved measuring the transparency of a block before and after annealing and extrapolating the block's absorption. After several instances of annealing, we discovered that the blocks had unique values of base, or undamaged, transparency. While further analysis is required, our conclusion at this time is that some of the blocks have experienced radiative damage that is irreparable by heat annealing. Nonetheless, the knowledge of deviations from a standard transparency in lead glass is vital in future applications.

7 Discussion

While we can presume that the calorimeter will produce effective results, due to the positive results observed in the annealing study, a continued study of annealing parameters and the limits of annealing efficacy would be greatly useful. The transparency loss in heated lead glass, as discovered and studied by Bogdan Wojtsekhowski and Fernando Torales-Acosta, is not well understood, and deserves further investigation. In addition, there is a question of whether the light guides and the epoxy used to adhere them to lead glass blocks will experience a similar phenomenon. A further study of transparency through these media at proposed annealing temperatures would provide a better prediction of uncertainty. Furthermore, computational analysis of the behavior of heat in order to extrapolate required heating parameters is made uncertain by the complexity of the C200 design. Current computational analysis conducted by Silviu Covrig uses a simplified model of the calorimeter and may only produce an approximation of the actual thermal system. Though this analysis will be pivotal in approximating a solution, its uncertainty should not be overlooked.

8 Acknowledgments

I would like to greatly acknowledge the assistance and guidance of Bogdan Wojtsekhowski, who made possible the realization of the various aforementioned studies. His role as my advisor and mentor at Jefferson Lab was pivotal in generating consistently outstanding experimental results. I would also like to acknowledge Albert Shahinyan, who supervised many of the outlined studies and provided many creative solutions, and Galust Sargsyan, who was the primary designer of the C200 calorimeter and who was always available to assist and supervise. Lastly, and foremost, I would like to acknowledge Konrad Aniol, who has granted my opportunity to intern at Jefferson Lab and has provided invaluable instruction as a research advisor.