# Candidate for the $2^{+}$excited Hoyle state at $E_{x} \sim 10 \mathrm{MeV}$ in ${ }^{12} \mathrm{C}$ 

M. Itoh, ${ }^{1}$ H. Akimune, ${ }^{2}$ M. Fujiwara, ${ }^{3}$ U. Garg, ${ }^{4}$ N. Hashimoto, ${ }^{3}$ T. Kawabata, ${ }^{5}$ K. Kawase, ${ }^{3}$ S. Kishi, ${ }^{5}$ T. Murakami, ${ }^{5}$ K. Nakanishi, ${ }^{3}$ Y. Nakatsugawa, ${ }^{5}$ B. K. Nayak, ${ }^{4}$ S. Okumura, ${ }^{3}$ H. Sakaguchi, ${ }^{3}$ H. Takeda, ${ }^{6}$ S. Terashima, ${ }^{5}$ M. Uchida, ${ }^{7}$ Y. Yasuda, ${ }^{3}$ M. Yosoi, ${ }^{3}$ and J. Zenihiro ${ }^{3}$<br>${ }^{1}$ Cyclotron and Radioisotope Center (CYRIC), Tohoku University, Sendai 980-8578, Japan<br>${ }^{2}$ Department of Physics, Konan University, Kobe 658-8501, Japan<br>${ }^{3}$ Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki, Osaka 567-0047, Japan<br>${ }^{4}$ Physics Department, University of Notre Dame, Notre Dame, Indiana 46556, USA<br>${ }^{5}$ Department of Physics, Kyoto University, Kyoto 606-8502, Japan<br>${ }^{6}$ RIKEN Nishina Center for Accelerator-Based Science, Wako, Saitama 351-0198, Japan<br>${ }^{7}$ Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, Japan

(Received 31 May 2011; published 14 November 2011)


#### Abstract

Inelastic scattering from ${ }^{12} \mathrm{C}$ has been measured at extremely forward angles including $0^{\circ}$ using $386 \mathrm{MeV} \alpha$ particles to study the $\alpha$-cluster states around $E_{x} \sim 10 \mathrm{MeV}$, especially the $2^{+}$state predicted by the $\alpha$-cluster model. We have analyzed ( $\alpha, \alpha^{\prime}$ ) cross-section data using both peak-fitting and multipole decomposition techniques. A $2^{+}$state at $E_{x}=9.84 \pm 0.06 \mathrm{MeV}$ with a width of $1.01 \pm 0.15 \mathrm{MeV}$ is found to be submerged in the broad $0^{+}$ state at $E_{x}=9.93 \pm 0.03 \mathrm{MeV}$ with a width of $2.71 \pm 0.08 \mathrm{MeV}$. This $2^{+}$state may be interpreted as the $2^{+}$ excitation of the Hoyle state and the $\alpha$-condensate state.


DOI: 10.1103/PhysRevC.84.054308
PACS number(s): 21.10.Re, 25.55.Ci, 25.70.Ef, 27.20.+n

## I. INTRODUCTION

The ${ }^{12} \mathrm{C}$ nucleus is among the most well-investigated nuclei in the nuclear chart. However, many unanswered questions concerning its nuclear structure still remain. Among them, a persistent question concerns the multipolarity of the broad level at $E_{x} \sim 10.3 \mathrm{MeV}$. In Ref. [1], this state has been tentatively assigned to $0^{+}$. According to the $3 \alpha$ resonating group method (RGM) calculation by Kamimura [2] and Uegaki et al. [3], there should be a $2^{+}$state around $E_{x} \sim$ 10 MeV as the $2^{+}$member of a $\beta$ band beginning with the $7.654 \mathrm{MeV} 0_{2}^{+}$level in ${ }^{12} \mathrm{C}$, since the coupling strength for the $2_{2}^{+} \rightarrow 0_{2}^{+}$transition is predicted to be 25 times larger than that for the $2_{1}^{+} \rightarrow 0_{1}^{+}$transition. These states have been predicted to be molecule-like states consisting of three $\alpha$ particles.

Tohsaki et al. suggested the $0_{2}^{+}$state can be a Bose-Einstein condensation-like state, in which all constituent $\alpha$-clusters condense into the lowest $S$-wave orbit [4,5]. If a $2^{+}$state indeed exists in the 10 MeV region, it might be an excited state of the Hoyle state and have a structure similar to the $0_{2}^{+}$ state in the $\alpha$-condensate model, in which one of the $\alpha$ clusters occupies a $D$-wave orbit [6-8]. Although these two pictures are very different, both calculations predicted the existence of a $2^{+}$state around 10 MeV , which was strongly coupled to the $7.654 \mathrm{MeV} \mathrm{O}_{2}^{+}$state. Furthermore, there should be a $0_{3}^{+}$state in the 10 MeV region, which can be considered as the vibrational mode along the broad energy surface for the $0_{2}^{+}$state according to the complex scaling method (CSM) calculation [9], or as the linear-like $3 \alpha$ structure in the antisymmetrized molecular dynamics (AMD) calculation [10].

In addition to the interest in its structure, the $0_{2}^{+}$state is very important from the viewpoint of nuclear astrophysics. Hoyle pointed out that the $0_{2}^{+}$state is a doorway state, which governs nuclear synthesis heavier than ${ }^{12} \mathrm{C}$ [11]. This state
is referred to as the Hoyle state. If an excited state exists closely coupled to the Hoyle state, the elemental abundance of nuclear matter in the universe may be affected. The Nuclear Astrophysics Compilation of Reaction Rates (NACRE) [12], which is widely used in astrophysical calculations, assumes the existence of the $2_{2}^{+}$state at $E_{x}=9.1 \mathrm{MeV}$. However, the exact location of the $2_{2}^{+}$state is experimentally unknown, leading to a large uncertainty in the reaction rate of $3 \alpha \rightarrow{ }^{12} \mathrm{C}$.

In terms of experiments, Jacquot et al. observed the ${ }^{12} \mathrm{C}\left(\alpha, \alpha^{\prime}\right)^{12} \mathrm{C}^{*}[3 \alpha]$ reaction by using $90 \mathrm{MeV} \alpha$ particles [13]. They claimed the 10.3 MeV state in ${ }^{12} \mathrm{C}$ was a $2^{+}$state by analyzing the momentum correlation between three $\alpha$ particles emitted therefrom. Motivated by the theoretical prediction in Ref. [2], Brandenburg et al. attempted to identify the spin and parity of the broad bump at $E_{x} \sim 10 \mathrm{MeV}$ [14]. However, this bump was dominated by the $0^{+}$component, and no significant $2^{+}$component was identified. John et al. measured inelastic $\alpha$ scattering from ${ }^{12} \mathrm{C}$ at $E_{\alpha}=240 \mathrm{MeV}$ [15]. They analyzed energy spectra by multipole decomposition analysis (MDA) and obtained isoscalar $E 0-E 4$ strengths up to $E_{x}=45 \mathrm{MeV}$. They reported a $2^{+}$component located at $E_{x}=11.46 \mathrm{MeV}$, which is slightly higher than the predicted excitation energy. In 2003, Itoh et al. reported the existence of the $2_{2}^{+}$state under the broad $0_{3}^{+}$bump at $E_{x} \sim 10 \mathrm{MeV}$ [16]. After this report, many experimentalists were eager to find out the $2_{2}^{+}$state in this excitation-energy region. Recently, Freer et al. measured inelastic proton scattering off ${ }^{12} \mathrm{C}$ at 66 and 200 MeV [17]. In their analysis, they included an additional $2^{+}$state with the width of 600 keV at $E_{x}=9.6 \mathrm{MeV}$ to explain the energy spectrum at $16^{\circ}$ where the cross section for the $0^{+}$state is small in the angular distribution. Muñoz-Britton et al. measured the angular correlation for the $3 \alpha$ decay in the ${ }^{12} \mathrm{C}\left({ }^{12} \mathrm{C},{ }^{12} \mathrm{C}[3 \alpha]\right)$ reaction [18]. However, no conclusive result for the $2_{2}^{+}$state was obtained.

Fynbo et al. examined the spin and parity of the excited states in ${ }^{12} \mathrm{C}$ fed by the $\beta$ decay of ${ }^{12} \mathrm{~B}$ and ${ }^{12} \mathrm{~N}$ [19]. They showed that the energy spectra around $E_{x} \sim 10 \mathrm{MeV}$ is well reproduced by taking the interference between the $0_{2}^{+}$state at $E_{x}=7.654 \mathrm{MeV}$ and the $0_{3}^{+}$state at $E_{x}=11.2 \mathrm{MeV}$ into account in the $R$-matrix analysis, and no $2^{+}$state is necessary around $E_{x} \sim 10 \mathrm{MeV}$ to explain their data. Recently, the same group obtained data with improved statistics and revised their $R$-matrix analysis [20]. They showed the existence of the broad 11.2 $\mathrm{MeV}^{+}$and $11.1 \mathrm{MeV} \mathrm{2}^{+}$states.

Very recently, M. Gai et al. performed the ${ }^{12} \mathrm{C}(\gamma, 3 \alpha)$ experiment at the High Intensity Gamma-Ray Source (HIGS) facility at Duke University [21]. They measured three decay $\alpha^{\prime}$ s with an optical readout time projection chamber (O-TPC) and observed a pure $E 2$ angular distribution most likely arising from a $2^{+}$state below 10 MeV . Although their preliminary result showed the existence of a $2^{+}$state, to date they have not reported its energy and width.

The experimental situation concerning the $2_{2}^{+}$state in ${ }^{12} \mathrm{C}$ is, thus, still controversial, and it is highly desirable to obtain conclusive evidence for the $2_{2}^{+}$state. In this paper, we report on two kinds of alternate analyses on inelastic $\alpha$-scattering data we had measured precisely for ${ }^{12} \mathrm{C}$ nucleus at $E_{\alpha}=386 \mathrm{MeV}$. One of them is peak-fitting analysis to ensure the existence of the $2^{+}$state and to obtain its strength. The other is MDA with a more realistic density predicted by the $\alpha$-cluster model than the previously adopted one in order to extract the reliable strength distributions of the $2^{+}$state and the broad $0^{+}$state. We find clear evidence for the second $2^{+}$state at $E_{x}=9.84 \pm 0.06 \mathrm{MeV}$.

## II. EXPERIMENT

The experiment was performed at the ring cyclotron facility of the Research Center for Nuclear Physics (RCNP), Osaka University, using the Grand Raiden spectrometer [22]. Details of the experimental setup and procedure are described in Ref. [23], and only a brief outline is provided below.

Inelastic scattering of $386 \mathrm{MeV} \alpha$ particles off ${ }^{12} \mathrm{C}$ has been measured at forward angles between $\theta=0^{\circ}$ and $15^{\circ}$. "Background-free" inelastic-scattering spectra were obtained at all angles, including $0^{\circ}$. A self-supporting natural carbon foil with a thickness of $2.84 \mathrm{mg} / \mathrm{cm}^{2}$ was used. The target foil contained oxygen and hydrogen (about 3\%) from the glue that was used in the preparation of the target. The contribution from the oxygen contaminant was estimated using ${ }^{28} \mathrm{Si}$ and $\mathrm{SiO}_{2}$ data and subtracted from the energy spectra. The data at crossover angles between inelastic scattering from ${ }^{12} \mathrm{C}$ and elastic scattering from hydrogen were not included in the analysis.

In the normal magnetic field setting of the Grand Raiden, particles scattered from the target are focused vertically and horizontally at the focal plane. On the other hand, instrumental background events due to rescattering of $\alpha$ particles on the wall and pole surfaces of the spectrometer are not focused in the vertical direction. Thus, we obtained background-free spectra by subtracting events at the off-median plane from those at the median plane. The energy spectra were measured in the range of $7 \leqslant E_{x} \leqslant 30 \mathrm{MeV}$ at $0^{\circ}$ and $3 \leqslant E_{x} \leqslant 30 \mathrm{MeV}$ at $2^{\circ}-15^{\circ}$.


FIG. 1. Energy spectra for the ${ }^{12} \mathrm{C}\left(\alpha, \alpha^{\prime}\right)$ reaction at scattering angles (a) $\theta_{\text {lab }}=0^{\circ}$ and (b) $\theta_{\text {lab }}=3.7^{\circ}$. The momentum acceptances of the spectrometer were in the range of $3 \leqslant E_{x} \leqslant 30 \mathrm{MeV}$ at $2.0^{\circ}-$ $15^{\circ}$ and of $7 \leqslant E_{x} \leqslant 30 \mathrm{MeV}$ at $0^{\circ}$, respectively.

Figure 1 shows typical energy spectra for the ${ }^{12} \mathrm{C}\left(\alpha, \alpha^{\prime}\right)$ reaction at $\theta_{\text {lab }}=0^{\circ}$ and $3.7^{\circ}$, where differential cross sections are at maximum for angular momentum transfer $L=0$ and 2, respectively. At $0^{\circ}$, the most prominent peak is the $0_{2}^{+}$state at $E_{x}=7.654 \mathrm{MeV}$. The broad bump at $E_{x} \sim 10 \mathrm{MeV}$ is also observed underneath the sharp $3_{1}^{-}$state at $E_{x}=9.641 \mathrm{MeV}$. Since the cross section for the $L=0$ transition becomes quite small at $3.7^{\circ}$, the $0_{2}^{+}$peak is almost invisible on the tail of the hydrogen contaminant bump. The broad bump around $E_{x} \sim$ 10 MeV also becomes small because this bump is dominated by the $0^{+}$component. However, a sizable strength remains around $E_{x} \sim 10 \mathrm{MeV}$. It suggests the broad bump contains higher multipole components with $L \neq 0$. To investigate the states at $E_{x} \sim 10 \mathrm{MeV}$ by means of MDA with a small energy-bin size, special care was exercised to keep the energy resolution of the beam stable. The energy resolution was about 200 keV through all runs.

Elastic scattering from ${ }^{12} \mathrm{C}$ was also measured at $\theta_{\text {c.m. }}=$ $4^{\circ}-35^{\circ}$ to determine the phenomenological $\mathrm{N}-\alpha$ interaction parameters with the same incident energy. The thickness of the carbon-graphite target for the measurement of elastic scattering was $30 \mathrm{mg} / \mathrm{cm}^{2}$.

## III. DISTORTED-WAVE BORN APPROXIMATION CALCULATION

Distorted-wave Born approximation (DWBA) calculations were carried out in the framework of the single-folding model with a density-dependent effective $\mathrm{N}-\alpha$ interaction [24] to obtain the angular distributions for various multipole components. The density-dependent effective $\mathrm{N}-\alpha$ interaction was given as

$$
\begin{align*}
V\left(\left|\mathbf{r}-\mathbf{r}^{\prime}\right|, \rho_{0}\left(r^{\prime}\right)\right) & =-V\left[1+\beta_{V} \rho_{0}\left(r^{\prime}\right)^{2 / 3}\right] \exp \left(-\left|\mathbf{r}-\mathbf{r}^{\prime}\right|^{2} / \alpha_{V}\right) \\
& -i W\left[1+\beta_{W} \rho_{0}\left(r^{\prime}\right)^{2 / 3}\right] \exp \left(-\left|\mathbf{r}-\mathbf{r}^{\prime}\right|^{2} / \alpha_{W}\right) \tag{1}
\end{align*}
$$

The parameters, $V=36.73 \mathrm{MeV}, W=25.9 \mathrm{MeV}$, and $\alpha_{V, W}=$ $3.7 \mathrm{fm}^{2}$ were obtained by fitting the measured elastic scattering angular distribution, as shown in Fig. 2. The density-dependent coefficients, $\beta_{V, W}=-1.9 \mathrm{fm}^{2}$, were taken from Ref. [25]. The ground-state density $\rho_{0}(r)$ was obtained by unfolding from the charge density measured by electron scattering [26] and the nucleon form factor [27]. The calculations were performed by using the code ECIS95 [28] with external form factors obtained


FIG. 2. Angular distribution of elastic scattering from ${ }^{12} \mathrm{C}$ at $E_{\alpha}=386 \mathrm{MeV}$. The solid line shows the result of the DWBA calculation using the single-folding model with the effective $\mathrm{N}-\alpha$ interaction.
by three models: the collective model [25,29]; the $3 \alpha$ RGM model [2]; and the $\alpha$-condensate model [30]. Figure 3 shows the angular distributions of (a) the $4.44 \mathrm{MeV}_{1}^{+}$, (b) the $7.65 \mathrm{MeV} 0_{2}^{+}$, and (c) the $9.64 \mathrm{MeV} \mathrm{3}_{1}^{-}$states. Yields of the $0_{2}^{+}$and the $3_{1}^{-}$states were extracted from the peak-fitting analysis explained in the next section. Absolute values of the angular distributions using collective transition densities are fitted to the experimental data. Although all of the calculations reproduced the experimental data up to $10^{\circ}$ quite well, the fits corresponding to the $3 \alpha$ RGM model were better than those from the collective model.

The reduced electric transition rates, $B(E L)$, are obtained from the $2^{L}$-pole transition moment as

$$
\begin{equation*}
B(E L)=\left|\frac{Z}{A} \int \rho_{L}^{\operatorname{tr}}(r) r^{L+2} d r\right|^{2} e^{2} \quad(L \geqslant 2) \tag{2}
\end{equation*}
$$

where $A, Z$, and $\rho_{L}^{\mathrm{tr}}(r)$ are the mass number, the atomic number, and the transition density for the angular momentum transfer, $L$, respectively. The $B(E 2)$ value of the $2_{1}^{+}$state obtained by the collective transition density is $37 \pm 1 e^{2} \mathrm{fm}^{4}$. The $B(E 3)$ value of the $3_{1}^{-}$state is $251 \pm 10 e^{2} \mathrm{fm}^{6}$. These values are in good agreement with those obtained by inelastic $\alpha$ scattering at 240 MeV [15]. In the case of the $3 \alpha$ RGM calculation, normalization is needed for the $0_{2}^{+}$and the $3_{1}^{-}$ states, respectively, as shown by the dashed lines in Fig. 3. The calculation reproduces angular distributions of the $0_{2}^{+}$and the $3_{1}^{-}$states up to $10^{\circ}$. The calculation for the $2_{1}^{+}$state reproduces the experimental data quantitatively.

## IV. PEAK-FITTING ANALYSIS

In order to confirm the existence of the $2^{+}$component, we performed a peak-fitting analysis of the excitation-energy spectra. In the $E_{x} \sim 10 \mathrm{MeV}$ region, there are several known states. Each of the $7.65 \mathrm{MeV} 0_{2}^{+}$and the $9.64 \mathrm{MeV} \mathrm{3}_{1}^{-}$states was fitted with two Gaussian functions to obtain better results for the fits. Since their intrinsic widths were smaller than


FIG. 3. Angular distributions of inelastic $\alpha$ scattering to (a) the $4.44 \mathrm{MeV}_{1}^{+}$state, (b) the $7.65 \mathrm{MeV}_{2}^{+}$state, and (c) the 9.64 MeV $3_{1}^{-}$state at $E_{\alpha}=386 \mathrm{MeV}$. The solid and dotted lines show the result of the DWBA calculation with the $3 \alpha$ RGM and collective models, respectively. The dashed lines show the $3 \alpha$ RGM calculations normalized to the experimental data.
the energy resolution of this experiment, their peak shapes reflect the structure of the beam. Broad peaks, such as the $10.3 \mathrm{MeV}_{3}^{+}$and the $10.84 \mathrm{MeV}_{1}^{-}$, were fitted with a single Gaussian function. An additional peak around 8.5 MeV was needed to fit energy spectra at $\theta_{\text {lab }}=0^{\circ}, 1.9^{\circ}$, and $2.3^{\circ}$, as shown in Fig. 4. We have established that this additional peak does not come from the oxygen contaminant. The contribution from the oxygen contaminant has already been subtracted, as described in Sec. II. Furthermore, the unnatural-parity $2^{-}$state at $E_{x}=8.87 \mathrm{MeV}$ in ${ }^{16} \mathrm{O}$ would be excited only very weakly, at best, by the spin-0 ( $\alpha, \alpha^{\prime}$ ) reaction. Therefore, the yield of this additional peak of 8.5 MeV was added to that of the broad bump. From the broad-bump spectrum, shown in Fig. 4, we subtracted the yields of the well-known states- $7.65 \mathrm{MeV} 0_{2}^{+}$, $9.64 \mathrm{MeV} \mathrm{3}_{1}^{-}$, and $10.84 \mathrm{MeV} \mathrm{1}_{1}^{-}$-using peak-fitting analysis and, thus, obtained the yield of the broad bump itself.

Figure 5 shows the angular distribution of the broad bump, which is fitted with the $L=0$ and $L=2$ angular distributions. The absolute differential cross sections for these transitions were arranged to explain the experimental angular distribution as follows:

$$
\begin{equation*}
\sigma^{\exp }(\theta)=\sum_{L=0,2} a_{L} \sigma_{L}^{\text {calc }}(\theta) \tag{3}
\end{equation*}
$$

The $L=0$ and $L=2$ angular distributions were calculated by using the transition density of the collective and $\alpha$ condensed model [30], respectively. Reduced $\chi^{2}$ of the fit to the experimental angular distribution is 75 . If we fit this experimental angular distribution without the $L=0$ or $L=$ 2 transitions, the reduced $\chi^{2}$ are 819 and 1097, respectively. If we use the $L=2$ angular distribution calculated by the


FIG. 4. Excitation-energy spectrum for ${ }^{12} \mathrm{C}$ fitted to the 7.65 MeV $0_{2}^{+}, 9.64 \mathrm{MeV} \mathrm{3}_{1}^{-}, 10.84 \mathrm{MeV} 1_{1}^{-}$, and the 10 MeV broad bump at $0^{\circ}$. The thick line shows the fitting result. The thin solid lines show the individual peaks. The additional peak around 8.5 MeV was needed to fit at $0^{\circ}, 1.9^{\circ}$, and $2.3^{\circ}$.
collective model, the reduced $\chi^{2}$ becomes worse. In order to estimate contributions from other multipole components, we also fitted the angular distribution with the $L=0,1,2$, 3 transitions. There was no $L=3$ contribution. However, a small contribution for the $L=1$ transition might remain in the broad bump. This could be caused by a failure to fit the broad bump with a single Gaussian function. However, since it was less than $6 \%$ at the maximum angle for the $L=1$ contribution, the existence of the $2^{+}$state together with the broad $0^{+}$state may be inferred. The $B(E 2)$ value obtained from the transition density of the $\alpha$-condensed model is $1.83 \pm 0.09 e^{2} \mathrm{fm}^{4}$. The $E 0$ strength of the $0_{2}^{+}$state obtained by inelastic $\alpha$ scattering was very sensitive to the interaction parameters and the transition densities and was significantly smaller than that obtained by the $\left(e, e^{\prime}\right)$ data [31].


FIG. 5. Angular distribution of inelastic $\alpha$ scattering to the broad bump around 10 MeV . The thin solid and dashed lines show the angular distributions calculated using the collective $(L=0)$ and the $\alpha$ condensed model [30] ( $L=2$ ) transition densities, respectively. The thick solid line shows the sum of the $L=0$ and 2 angular distributions. The dot-dashed line shows the possible $L=1$ component.

We, therefore, show just the ratio of the $E 0$ strength between $B\left(E 0 ; 0_{1} \rightarrow 0_{3}\right)$ and $B\left(E 0 ; 0_{1} \rightarrow 0_{2}\right)$ obtained from the same model. The $B(E 0)$ value is obtained as follows:

$$
\begin{equation*}
B(E 0)=\left|\frac{Z}{A} \int \rho_{0}^{\operatorname{tr}}(r) r^{4} d r\right|^{2} e^{2} \tag{4}
\end{equation*}
$$

The $B\left(E 0 ; 0_{1} \rightarrow 0_{3}\right) / B\left(E 0 ; 0_{1} \rightarrow 0_{2}\right)$ obtained by the collective and the $3 \alpha$ RGM models are 1.0 and 0.84 , respectively.

## V. MULTIPOLE DECOMPOSITION ANALYSIS

In order to find the strength distributions for the $0_{3}^{+}$and the $2_{2}^{+}$states, we performed MDA with the small energy-bin size. The angular distributions of the double-differential cross sections were obtained by dividing the energy spectrum into $0.25-\mathrm{MeV}$ bins and sorting in terms of scattering angles. Since the DWBA calculation reproduces well the experimental angular distributions for the $2_{1}^{+}$, the $0_{2}^{+}$, and the $3_{1}^{-}$states up to $10^{\circ}$ as shown in the previous section, we used the experimental data up to $10^{\circ}$ in the MDA. Inelastic $\alpha$ scattering has a selectivity for the isoscalar natural-parity transition, and its angular distributions are characterized by the transferred angular momentum $L$. In MDA, the experimentally obtained cross sections are expressed as the sum of the contributions from the various multipole components as

$$
\begin{equation*}
\sigma^{\exp }\left(\theta, E_{x}\right)=\sum_{L} a_{L}\left(E_{x}\right) \sigma_{L}^{\mathrm{calc}}\left(\theta, E_{x}\right) \tag{5}
\end{equation*}
$$

where $E_{x}, \theta$ are the excitation energy and the scattering angle, respectively, and $\sigma_{L}^{\text {calc }}\left(\theta, E_{x}\right)$ is the DWBA cross section for the transferred angular momentum $L$. Multipole components up to $L=5$ were taken into account in the fit, since the first maximum of the angular distribution for the $L=5$ transfer appears at $10^{\circ}$. To obtain a better fit with the MDA, the angular distributions for $L=0$ and 3 were calculated by the $3 \alpha$ RGM model, and that for $L=2$ was calculated by the $\alpha$-condensate model [30]. Those for other multipole transitions were calculated by the collective model [25,29]. In the MDA, the shape of the strength distribution is roughly determined by $L$. However, since the differences between the transition densities for the collective and cluster models are large, the fits are better for the chosen models.

Figure 6 shows the angular distribution for each energy bin of 0.25 MeV . The solid lines show the fits to the experimental data. They reproduce the angular distribution of each energy bin very well. Figure 7 shows the energy spectra and the results of the MDA at (a), (c) $\theta_{\text {lab }}=0^{\circ}$, and (b), (d) $\theta_{\text {lab }}=$ $3.7^{\circ}$, respectively. The broad bump at $0^{\circ}$ is dominated by the $L=0$ component. On the other hand, that at $\theta_{\text {lab }}=3.7^{\circ}$ is dominated by the $L=2$ component. Figure 8 shows the isoscalar-strength distributions for the $L=0,1,2$, and 3 transitions. The isoscalar-strength distributions were extracted from the following equations:

$$
\begin{gather*}
S_{L}\left(E_{x}\right)=a_{L}\left(E_{x}\right)\left|\int \rho_{L}^{\mathrm{tr}}\left(r, E_{x}\right) r^{L+4} d r\right|^{2}(L=0,1)  \tag{6}\\
S_{L}\left(E_{x}\right)=a_{L}\left(E_{x}\right)\left|\int \rho_{L}^{\operatorname{tr}}\left(r, E_{x}\right) r^{L+2} d r\right|^{2}(L \geqslant 2) \tag{7}
\end{gather*}
$$



FIG. 6. Angular distributions of the double-differential cross section at $E_{x}=8-12 \mathrm{MeV}$ for various excitation-energy bins in ${ }^{12} \mathrm{C}$. The thick solid lines show the fits to the data from multipole decomposition. In each panel, the contributions from $L=0$ (thin solid), $L=1$ (dot-dashed), $L=2$ (dashed) and $L=3$ (dotted) are also displayed. Contributions from other multipoles are not displayed.

In the calculations with the $3 \alpha$ RGM model and the $\alpha$ condensate model, we assumed that there were no excitationenergy dependences of the transition densities. The wellknown $3_{1}^{-}$state at $E_{x}=9.64 \mathrm{MeV}$ and the $1_{1}^{-}$state at $E_{x}=$ 10.84 MeV are clearly seen in Figs. 8(d) and 8(b), respectively. In addition to these, one sees the broad $0^{+}$strength at $E_{x}=$ $9.93 \pm 0.03 \mathrm{MeV}$ with a width of $2.71 \pm 0.08 \mathrm{MeV}$, and the $2^{+}$strength at $E_{x}=9.84 \pm 0.06 \mathrm{MeV}$ with a width of $1.01 \pm 0.15 \mathrm{MeV}$, even though the MDA uncertainties in the $L=0$ and $L=2$ strengths are large at $E_{x}=9.64 \mathrm{MeV}$ because of the strong $3_{1}^{-}$state at about the same energy. Following conventional procedures, the positions and widths of these states were obtained by fitting each with a single Gaussian function. The $2^{+}$state, predicted by several theories [2,3,6,9], has been confirmed. The $B(E 2)$ value obtained by integrating the $L=2$ strength distribution from 9 to 11 MeV and multiplying a factor of $e^{2} / 4$ is $1.6 \pm 0.2 e^{2} \mathrm{fm}^{4}$. This value is consistent with the result of the peak-fitting analysis reported earlier in the paper.


FIG. 7. Excitation-energy spectra from ${ }^{12} \mathrm{C}\left(\alpha, \alpha^{\prime}\right)$ at $\theta_{\text {lab }}=0^{\circ}$, $3.7^{\circ}$. The hatched regions were constructed from the results of the MDA. The cross-hatched region is $L=0$, the right-hatched is $L=2$, the left-hatched is $L=3$, and the vertical-hatched regions represent contributions from other multipoles.

## VI. DISCUSSION

The newly found $2_{2}^{+}$state is located at $E_{x}=$ $9.84 \pm 0.06 \mathrm{MeV}$ with a width of $1.01 \pm 0.15 \mathrm{MeV}$, both values close to those predicted by many $\alpha$-cluster model calculations [2,3,6,9], using $2^{+}$wave functions strongly coupled to the Hoyle state. This correspondence strongly suggests that the $2_{2}^{+}$state has a highly developed $3 \alpha$ structure, and is inferred to be an excited state of the Hoyle state. It is noted that the existence of the $2_{2}^{+}$state at 9.6 MeV in ${ }^{12} \mathrm{C}$ has been discussed by Zimmerman et al. [32]. This $2_{2}^{+}$state at 9.6 MeV would correspond to the $9.84 \mathrm{MeV} 2_{2}^{+}$state in ${ }^{12} \mathrm{C}$, which is reported in this paper. The astrophysical NACRE [12]


FIG. 8. Isoscalar-strength distributions for (a) $L=0$, (b) $L=1$, (c) $L=2$, and (d) $L=3$ as obtained in this paper. The vertical axes show the corresponding isoscalar strengths. The solid lines show the conventional fit by a Gaussian function in (a) and (c). The dashed line in (a) shows the fit by two Gaussian functions.
includes the existence of the $2^{+}$state at $E_{x}=9.1 \mathrm{MeV}$, which has not been experimentally observed. Fynbo et al. [19] had excluded the $9.1 \mathrm{MeV} 2^{+}$contribution according to their $\beta$-decay experiment. However, since the strength distribution for the $2_{2}^{+}$state obtained in our experiment rises from $E_{x} \sim$ 9 MeV as shown in Fig. 8(c), it is imperative that astrophysical calculations include effects of this broad $2_{2}^{+}$state.

The $2^{+}$state at 11.46 MeV , which was reported in Ref. [15], was not observed in the present experiment. Neither could we discern any peaks corresponding to the $0^{+}$state at $E_{x}=$ 11.2 MeV or the $2^{+}$state at $E_{x}=11.1 \mathrm{MeV}$, reported in Ref. [20]. On the other hand, the broad $0_{3}^{+}$state at $E_{x}=$ $9.93 \pm 0.03 \mathrm{MeV}$ with a width of $2.71 \pm 0.08 \mathrm{MeV}$ is in good agreement with that measured in inelastic $\alpha$ scattering at $E_{\alpha}=240 \mathrm{MeV}$ [15]. However, the strength distribution has an asymmetric shape, which may be due to two different $0^{+}$ states. As shown in Fig. 8(a), these two states are located at $E_{x}=9.04 \pm 0.09 \mathrm{MeV}$ with a width of $1.45 \pm 0.18 \mathrm{MeV}$ and at $E_{x}=10.56 \pm 0.06 \mathrm{MeV}$ with a width of $1.42 \pm 0.08 \mathrm{MeV}$, respectively, and may correspond to the $0_{3}^{+}$and $0_{4}^{+}$states described in Ref. [9]. Further investigations of the microscopic structure of these states are needed, both experimentally and theoretically.

## VII. CONCLUSION

We have measured inelastic scattering of $\alpha$ particles at $E_{\alpha}=386 \mathrm{MeV}$ from ${ }^{12} \mathrm{C}$. The angular distributions of the
differential cross sections from $\theta=0^{\circ}$ to $10^{\circ}$ and from $E_{x}=3$ to 30 MeV were obtained. By using a peak-fitting analysis, the angular distribution for the broad bump around $E_{x}=10 \mathrm{MeV}$ was extracted and analyzed using a DWBA calculation. The strength distribution for the $L=0,1,2,3$ transitions was extracted by the MDA. As a result, a broad $0_{3}^{+}$state at $E_{x}=$ $9.93 \pm 0.03 \mathrm{MeV}$ with a width of $2.71 \pm 0.08 \mathrm{MeV}$ and a broad $2_{2}^{+}$state at $E_{x}=9.84 \pm 0.06 \mathrm{MeV}$ with a width of $1.01 \pm 0.15 \mathrm{MeV}$ were clearly identified. This $2_{2}^{+}$state is a good candidate for the excited state of the Hoyle resonance and also the $\alpha$-particle condensate state.

## ACKNOWLEDGMENTS

The authors acknowledge RCNP cyclotron staff for providing a very stable and high-quality beam for the $0^{\circ}$ and extremely forward angle measurements. The authors also acknowledge Dr. Y. Funaki for providing the transition density of the $2_{2}^{+}$state and for stimulating discussions. Thanks are also due to Profs. H. Horiuchi and A. Tohsaki for their interest and for many discussions concerning this experiment. This work was performed under the RCNP E200 program and supported in part by Grant Aid for Scientific Research No. 19740119 from the Japanese Ministry of Education, Sports, Culture, Science, and Technology and the United States National Science Foundation (Grants No. INT03-42942, PHY07-58100, and PHY-1068192).
[1] F. Ajzenberg-Selove, Nucl. Phys. A 506, 1 (1990).
[2] M. Kamimura, Nucl. Phys. A 351, 456 (1981).
[3] E. Uegaki, S. Okabe, Y. Abe, and H. Tanaka, Prog. Theor. Phys. 57, 1262 (1977).
[4] A. Tohsaki, H. Horiuchi, P. Schuck, and G. Röpke, Phys. Rev. Lett. 87, 192501 (2001).
[5] H. Horiuchi, in Clusters in Nuclei, Lecture Notes in Physics Volume 818, edited by C. Beck (Springer-Verlag, Berlin, 2010), p. 57.
[6] Y. Funaki, A. Tohsaki, H. Horiuchi, P. Schuck, and G. Röpke, Eur. Phys. J. A 24, 321 (2005).
[7] T. Yamada and P. Schuck, Phys. Rev. C 69, 024309 (2004).
[8] T. Yamada et al., in Clusters in Nuclei, Lecture Notes in Physics, edited by C. Beck, Vol. 2 (Springer-Verlag, Berlin, 2010).
[9] C. Kurokawa and K. Kato, Nucl. Phys. A 792, 87 (2007).
[10] Y. Kanada-En'yo, Prog. Theor. Phys. 117, 655 (2007); Y. Kanada-En'yo and M. Kimura, in Clusters in Nuclei, Lecture Notes in Physics Volume 818, edited by C. Beck (Springer-Verlag, Berlin, 2010), p. 129.
[11] F. Hoyle, Astrophys. J. Suppl. Ser. 1, 121 (1954).
[12] C. Angulo et al., Nucl. Phys. A 656, 3 (1999).
[13] C. Jacquot, Y. Sakamoto, M. Jung, and L. Girardin, Nucl. Phys. A 201, 247 (1973).
[14] S. Brandenburg, A. Drentje, M. Harakeh, and A. van der Woude, KVI Annual Report, 1985 (unpublished).
[15] B. John, Y. Tokimoto, Y.-W. Lui, H. L. Clark, X. Chen, and D. H. Youngblood, Phys. Rev. C 68, 014305 (2003).
[16] M. Itoh et al., Nucl. Phys. A 738, 268 (2004).
[17] M. Freer et al., Phys. Rev. C 80, 041303(R) (2009).
[18] T. Muñoz-Britton et al., J. Phys. G 37, 105104 (2010).
[19] H. O. U. Fynbo et al., Nature (London) 433, 136 (2005).
[20] S. Hyldegaard et al., Phys. Rev. C 81, 024303 (2010).
[21] M. Gai for the UConn-Yale-Duke-Weizmann-PTB-UCL Collaboration, J. Phys. C 267, 012046 (2011); for the UConn-Yale-Duke-Weizmann-PTB-UCL Collaboration, Acta Phys. Pol. B 42, 775 (2011).
[22] M. Fujiwara et al., Nucl. Instrum. Methods Phys. Res., Sect. A 422, 484 (1999).
[23] M. Itoh et al., Phys. Rev. C 68, 064602 (2003).
[24] A. Kolomiets, O. Pochivalov, and S. Shlomo, Phys. Rev. C 61, 034312 (2000).
[25] G. R. Satchler, Nucl. Phys. A 472, 215 (1987).
[26] H. De Vries et al., At. Data Nucl. Data Tables 36, 495 (1987).
[27] H. Sakaguchi, Memoirs of the Faculty of Science, Kyoto University, Series of Physics, Astrophysics, Geophysics and Chemistry 36, Article 4 (1982).
[28] J. Raynal, computer code, ECIS95, NEA0850-14.
[29] M. Harakeh and A. van der Wounde, Giant Resonances (Clarendon, Oxford, 2001).
[30] Y. Funaki, A. Tohsaki, H. Horiuchi, P. Schuck, and G. Röpke, Eur. Phys. J. A 28, 259 (2006).
[31] P. Strehl, Z. Phys. 234, 416 (1970).
[32] W. R. Zimmerman, N. E. Destefano, M. Freer, M. Gai, and F. D. Smit, Phys. Rev. C 84, 027304 (2011).

