Collective Nuclear Motions

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The Shell model provides a basis for all nuclear models but its application is cleanest near closed shells of N and Z. The reason for this is that a closed shell is a particularly stable configuration, energetically speaking, since the protons and neutrons have paired off to zero total angular momentum. A single nucleon, proton or neutron, above the closed shell energy gap can jump between unfilled states above the closed shell and thus excited states of the system are dominated by the single nucleon excitation.

Between closed shells nucleons can can exist in orbits that are relatively closely spaced in energy and there is flexibility for multi nucleon excitations in the orbits. These multi nucleon excitations are called collective excitations. There are two broad categories of collective excitations: Rotational and Vibrational excitations.

1 Rotational Models

Here the spectrum of excited states has the characteristic of a rotating body. Classically we can define the kinetic energy, E, of rotation of a body of moment of inertia I and angular momentum L as

$$E(L) = \frac{L^2}{2I}. \tag{1}$$

Transcribing this expression into a quantum statement we have

$$E(L) = \frac{\hbar^2 L(L+1)}{2I}. \tag{2}$$

Thus the signature of collective rotational motion will be energy level spacing proportional to $L(L+1)$ and increasing in a regular manner from $L = 0, 2, 4, 6, 8, ...$ See figure 1.

2 Vibrational Models

If one considers the nucleus to be a liquid drop one can invoke the classical model of the vibrations of a liquid drop. The even-even nucleus is in its ground

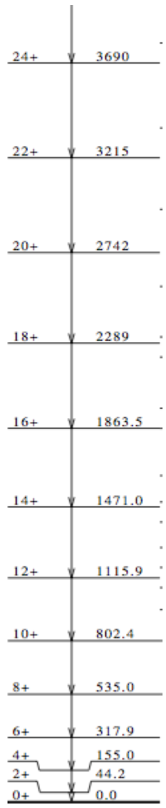

Figure 1: Energy level diagram for the low lying states of ^{244}Pu illustrating rotational structure of its excited states.

state with $J^\pi = 0^+$. The lowest energy vibration is a quadrupole phonon. A quadrupole phonon carries angular momentum $L = 2$. The next lowest energy would be the state carrying 2 quadrupole phonons. The possible angular momentum states are $2 + 2 = 0, 2, 4$ These three excited states with $J^\pi = 0^+, 2^+, 4^+$ would be energetically degenerate in the simplest model. In real nuclei these three states are often close together in energy, but not degenerate. A signature of a vibrational nucleus is a $J^\pi = 0^+$ ground state followed by a first excited 2^+ state at energy $E(2)$ and then a triplet of states of spin $0^+, 2^+, 4^+$ whose average energy is about $2E(2)$. See figure 2.

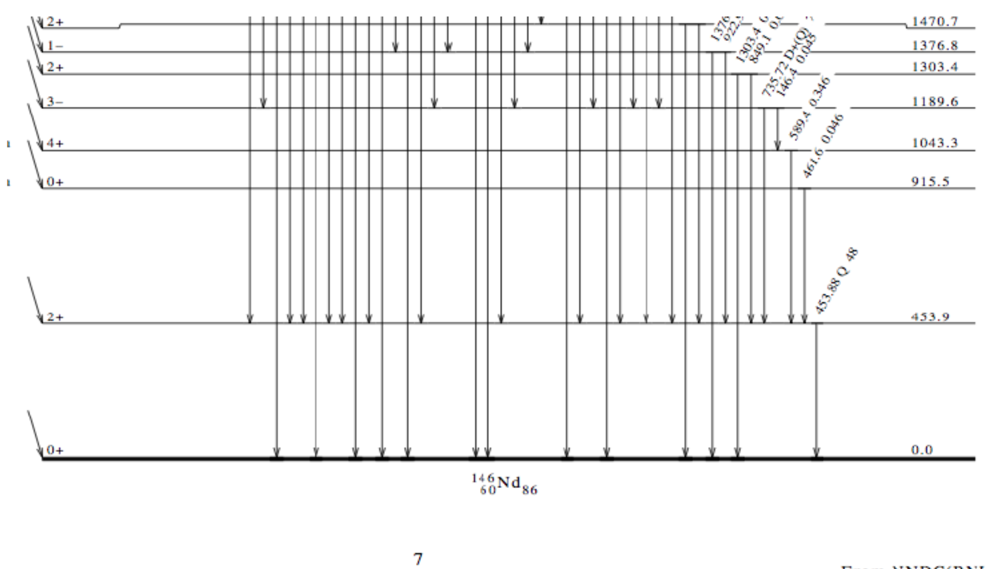

Figure 2: Energy level diagram for the low lying states of ^{146}Nd illustrating vibrational structure of its excited states.