

electromagnetic energy of the proton

From hadron spectroscopy we learn that the d quark is slightly more massive than the u quark. This accounts for the larger mass of the neutron compared to the proton. We expect that if the masses of the u and d quarks were equal the proton should be more massive than the neutron because it has a net charge.

Suppose the proton were a uniformly charged sphere of radius $1F$ and constant density,

$$\rho_0 = e/(4\pi R^3/3).$$

a) Find the mean square radius of the proton's charge distribution, defined as

$\langle r^2 \rangle = (1/e) \int_{vol} r^2 dq$, where $dq = \rho(r)dV$. Here dq is the element of charge located at r .

b) Find the Coulomb energy of this charge distribution. You can solve this analytically or numerically.

A more realistic charge distribution for the proton which gives a good description of the electron-proton elastic scattering cross section to Q^2 around and less than 1 GeV^2 is an exponential decay:

$$\rho(r) = e(\alpha^3/8\pi)\exp(-\alpha r), \text{ where } \alpha = 4.26F^{-1}.$$

d) Find the mean square radius of the proton's charge distribution, defined as

$\langle r^2 \rangle = (1/e) \int_{vol} r^2 dq$, where $dq = \rho(r)dV$. Here dq is the element of charge located at r .

e) Find the Coulomb energy of this charge distribution. You can solve this analytically or numerically.

f) What would be the physical consequences of a d quark equal in mass or slightly less massive than the u quark? Feel free to speculate!

Note: A useful representation of the Coulomb energy in units of MeV and F is $V_{Coul} = Ze^2/R$ and $e^2 = 1.44\text{MeV} \cdot F$.